ON THE BRAUER-SPEISER THEOREM

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Communicated by I. N. Herstein, October 1, 1970

Let $\chi$ be an absolutely irreducible rational valued character of a finite group $G$. The component of the group algebra $\mathbb{Q}G$ corresponding to $\chi$ is central simple over $\mathbb{Q}$ and the $\mathbb{Q}G$-irreducible module of this component affords the character $m_\mathbb{Q}(\chi)\chi$ which is also rational valued; hence this module is isomorphic to its dual, whence its endomorphism ring (i.e. the division algebra appearing in the simple component) is isomorphic to its opposite and so is a quaternion algebra (Albert-Hasse-Brauer-Nöether). This result is known as the Brauer-Speiser Theorem [1], [2].

We ask: Does every quaternion division algebra central simple over $\mathbb{Q}$ appear in some $\mathbb{Q}G$? The answer is yes: Let $G$ be generated by $x, y, c$ subject to the relations $x^p = 1$ ($p$ odd), $y^{-1}xy = x^a$ ($a$ is primitive mod $p$), $y^p = 1$, $c^2 = 1$ and $c$ central. Then $\mathbb{Q}G$ contains as a simple component the cyclic algebra $(\mathbb{Q}(\xi_p), (\tau), -1)$, which is c.s. over $\mathbb{Q}$ and has Hasse-invariant $1/2$ at $\mathfrak{A}$ and $p$. The quaternion group of order 8 yields the ordinary quaternion algebra (Hasse-invariant $1/2$ at $\mathfrak{A}$ and 2) and so, by taking tensor products, we see that every quaternion algebra is available.

REFERENCES


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AMS 1969 subject classifications. Primary 2080; Secondary 1640.

Key words and phrases. Dual, Hasse-invariant.

1 Research supported by The National Science Foundation.

2 Herstein and the author have shown that a simple component of $\mathbb{Q}G$ is descended from the maximal real subfield of its center if and only if it is of index at most 2 in the Brauer group. (To appear in J. Algebra.)

3 This result has also been obtained independently by Mark Benard.