BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 2, March 1971

ON THE BRAUER-SPEISER THEOREM[1]

BY K. L. FIELDS

Communicated by I. N. Herstein, October 1, 1970

Let $\mathfrak{X}$ be an absolutely irreducible rational valued character of a finite group G. The component of the group algebra QG corresponding to $\mathfrak{X}$ is central simple over Q and the QG-irreducible module of this component affords the character $m_Q(\mathfrak{X})\mathfrak{X}$ which is also rational valued; hence this module is isomorphic to its dual, whence its endomorphism ring (i.e. the division algebra appearing in the simple component) is isomorphic to its opposite and so is a quaternion algebra (Albert-Hasse-Brauer-Nöether). This result is known as the Brauer-Speiser Theorem [1], [2].[2]

We ask: Does every quaternion division algebra central simple over Q appear in some QG? The answer is yes: Let G be generated by x, y, c subject to the relations $x^p=1$ (p odd), $y^{-1}xy=x^a$ (a is primitive mod p), $y^{p-1}=c$, $c^2=1$ and c central. Then QG contains as a simple component the cyclic algebra $\langle Q(\xi_p), \langle\tau\rangle, -1\rangle$, which is c.s. over Q and has Hasse-invariant 1/2 at $\mathfrak{R}$ and p. The quaternion group of order 8 yields the ordinary quaternion algebra (Hasse-invariant 1/2 at $\mathfrak{R}$ and 2) and so, by taking tensor products, we see that every quaternion algebra is available.[3]

REFERENCES

1. R. Brauer, *Representation theory of finite groups*, Lectures on Modern Math., vol. 1, Wiley, New York, 1963. MR **31** #2314.

2. B. Fein, *Note on the Brauer-Speiser theorem*, Proc. Amer. Math. Soc. **25** (1970), 620–622.

UNIVERSITY OF CHICAGO, CHICAGO, ILLINOIS 60637

AMS 1969 *subject classifications*. Primary 2080; Secondary 1640.
Key words and phrases. Dual, Hasse-invariant.
[1] Research supported by The National Science Foundation.

[2] Herstein and the author have shown that a simple component of QG is descended from the maximal real subfield of its center if and only if it is of index at most 2 in the Brauer group. (To appear in J. Algebra.)

[3] This result has also been obtained independently by Mark Benard.