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A NORMAL SPACE X FOR WHICH $X \times I$ IS NOT NORMAL

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In [1] C. Dowker gave a number of interesting characterizations of normal Hausdorff spaces whose cartesian product with the closed unit interval is not normal. Thus, such a space is often called a Dowker space; a Dowker space X will be described below. It was previously known only that the existence of a Dowker space is consistent with the usual axioms of set theory [2], [3]. The proof that X is a Dowker space uses no set theoretic assumptions beyond the axiom of choice.

We use the convention that an ordinal λ is the set of all ordinals less than λ. An ordinal γ is said to be *cofinal* with λ if there is a subset Γ of λ order isomorphic with γ such that $\alpha \in \lambda$ implies $\alpha \leqq \beta$ for some $\beta \in \Gamma$; let $\operatorname{cf}(\lambda)$ be the smallest ordinal cofinal with λ.

Define $F = \{ f : \omega_0 \to \omega_\omega \mid \forall n \in \omega_0,\ f(n) \leqq \omega_n + 1 \}$.

Define $X = \{ f \in F \mid \exists k \in \omega_0 \text{ such that } \forall n \in \omega_0,\ \omega_0 < \operatorname{cf}(f(n)) < \omega_k \}$.

For f and g in F, define $U_{fg} = \{ h \in X \mid \forall n \in \omega_0,\ f(n) < h(n) \leqq g(n) \}$. Then topologize X by using the set of all U_{fg} for f and g in F as a basis for the topology. The resulting space is a collectionwise normal Dowker space.

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