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CODIMENSION-ONE FOLIATIONS OF SPHERES

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Let M be an n-dimensional, differentiable manifold with a (possibly empty) boundary ∂M. A *smooth, codimension-one foliation* of M is a decomposition of M into disjoint, connected subsets, called the *leaves* of the foliation, with the following properties:

(i) At each point $p \in M$ there exist local C^∞-coordinates $(x_1, \cdots, x_n)$ such that in a neighborhood of p the leaves are described by the equations $x_n = \text{constant}$.

(ii) Each component of ∂M is a leaf.

In 1951 George S. Reeb constructed a smooth, codimension-one foliation of S^3 [**4**], and it has since been shown by Lickorish [**2**], and independently by Novikov and Zieschang, that, in fact, every compact, orientable 3-manifold can be so foliated. By using the polynomial $p(Z_0, Z_1, Z_2) = Z_0^3 + Z_1^3 + Z_2^3$ in complex 3-space and the theorems in [**3**], we prove the following:

THEOREM 1. *There exists a smooth, codimension-one foliation of S^5 having one compact leaf B such that*:

(a) *B is diffeomorphic to $S^1 \times L$ where L is a circle bundle over a 2-torus, T^2.*

(b) *All the noncompact leaves of one component of the foliation are diffeomorphic to $R^2 \times T^2$.*

(c) *All the noncompact leaves of the other component have the homotopy-type of a bouquet $S^2 \vee \cdots \vee S^2$ of eight 2-spheres.*

By using Theorem 1 and an inductive procedure, we then establish

THEOREM 2. *There exist smooth, codimension-one foliations of each of the spheres S^{2^k+3} for $k = 1, 2, 3, \cdots$. (The sequence begins: S^5, S^7, S^{11}, S^{19}, S^{35}, $\cdots$.)*

COROLLARY 1. *For $n = 2^k + 1$, $k = 1, 2, 3, \cdots$, there exist smooth, codimension-one foliations of the manifolds $D^2 \times S^n$ and $D^2 \times V_{n+1,2}$ where $V_{n+1,2} = \mathrm{SO}(n+1)/\mathrm{SO}(n-1)$.*

COROLLARY 2. *For $n = 2^k + 4$, $k = 1, 2, 3, \cdots$, there exist smooth,*

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codimension-one foliations of the classical groups SO(n), SU$(n/2)$, Sp$(n/4)$ *and their associated Stiefel manifolds.* (*For the* Sp-*case we must have* $k>1$.)

Let $\mathbf{C}^{n+1}$ denote $(n+1)$-dimensional, complex number space and set

$$S^{2n+1} = \{Z \in \mathbf{C}^{n+1}: |Z|^2 = 1\}.$$

We consider, for each integer d, the compact, differentiable manifold

$$\Sigma^{2n-1}(d) = \{Z \in S^{2n+1}: Z_0^d + Z_1^2 + Z_2^2 + \cdots + Z_n^2 = 0\}\cdot$$

If $d \equiv \pm 1 \pmod 8$, then $\Sigma^{2n-1}(d)$ is a standard $(2n-1)$-sphere which is knotted in S^{2n+1} [**1**, §11]. Using Corollary 1 and [**3**, Theorem 4.8], we obtain

COROLLARY 3. *For* $n=2^{k-1}+1$, $k=2, 3, 4, \cdots$, *and for each* $d\equiv \pm 1 \pmod 8$ *there exists a smooth, codimension-one foliation of* S^{2n+1} *having as a compact leaf the boundary of a tubular neighborhood of the knotted sphere* $\Sigma^{2n-1}(d)$.

We then change our approach and study the natural action of SO(n) on $\Sigma^{2n-1}(d)$ (cf. [**1**, §5]). By working with the orbit space and using Corollary 1, we are able to prove

THEOREM 3. *For* $n=2^k+3$, $k=1, 2, 3, \cdots$, *and for any* d, *there exists a smooth, codimension-one foliation of the manifold* $\Sigma^n(d)$.

Corollary 1 is due to Alberto Verjovsky whose conversation was of great value to me during the preparation of this work. Detailed proofs of the above theorems will appear elsewhere.

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