Let \( M \) be an \( n \)-dimensional, differentiable manifold with a (possibly empty) boundary \( \partial M \). A smooth, codimension-one foliation of \( M \) is a decomposition of \( M \) into disjoint, connected subsets, called the leaves of the foliation, with the following properties:

(i) At each point \( p \in M \) there exist local \( C^\infty \)-coordinates \((x_1, \ldots, x_n)\) such that in a neighborhood of \( p \) the leaves are described by the equations \( x_n = \text{constant} \).

(ii) Each component of \( \partial M \) is a leaf.

In 1951 George S. Reeb constructed a smooth, codimension-one foliation of \( S^2 \) \([4]\), and it has since been shown by Lickorish \([2]\), and independently by Novikov and Zieschang, that, in fact, every compact, orientable 3-manifold can be so foliated. By using the polynomial \( p(Z_0, Z_1, Z_2) = Z_0^3 + Z_1^3 + Z_2^3 \) in complex 3-space and the theorems in \([3]\), we prove the following:

**Theorem 1.** There exists a smooth, codimension-one foliation of \( S^8 \) having one compact leaf \( B \) such that:

(a) \( B \) is diffeomorphic to \( S^1 \times L \) where \( L \) is a circle bundle over a 2-torus, \( T^2 \).

(b) All the noncompact leaves of one component of the foliation are diffeomorphic to \( \mathbb{R}^2 \times T^2 \).

(c) All the noncompact leaves of the other component have the homotopy-type of a bouquet \( S^2 \vee \cdots \vee S^2 \) of eight 2-spheres.

By using Theorem 1 and an inductive procedure, we then establish

**Theorem 2.** There exist smooth, codimension-one foliations of each of the spheres \( S^{4k+3} \) for \( k = 1, 2, 3, \ldots \). (The sequence begins: \( S^8, S^7, S^{11}, S^{19}, S^{38}, \ldots \).

**Corollary 1.** For \( n = 2^k + 1, k = 1, 2, 3, \ldots \), there exist smooth, codimension-one foliations of the manifolds \( D^2 \times S^n \) and \( D^2 \times V_{n+1,2} \) where \( V_{n+1,2} = \text{SO}(n+1)/\text{SO}(n-1) \).

**Corollary 2.** For \( n = 2^k + 4, k = 1, 2, 3, \ldots \), there exist smooth,
codimension-one foliations of the classical groups $\text{SO}(n)$, $\text{SU}(n/2)$, $\text{Sp}(n/4)$ and their associated Stiefel manifolds. (For the Sp-case we must have $k > 1$.)

Let $\mathbb{C}^{n+1}$ denote $(n+1)$-dimensional, complex number space and set

$$S^{2n+1} = \{ Z \in \mathbb{C}^{n+1}: |Z|^2 = 1 \}.$$

We consider, for each integer $d$, the compact, differentiable manifold

$$\Sigma^{2n-1}(d) = \{ Z \in S^{2n+1}: Z_{d}^{2} + Z_{1}^{2} + Z_{2}^{2} + \cdots + Z_{n}^{2} = 0 \}.$$

If $d \equiv \pm 1 \pmod{8}$, then $\Sigma^{2n-1}(d)$ is a standard $(2n-1)$-sphere which is knotted in $S^{2n+1}$ [1, §11]. Using Corollary 1 and [3, Theorem 4.8], we obtain

**Corollary 3.** For $n = 2k-1 + 1$, $k = 2, 3, 4, \ldots$, and for each $d \equiv \pm 1 \pmod{8}$ there exists a smooth, codimension-one foliation of $\Sigma^{2n+1}$ having as a compact leaf the boundary of a tubular neighborhood of the knotted sphere $\Sigma^{2n-1}(d)$.

We then change our approach and study the natural action of $\text{SO}(n)$ on $\Sigma^{2n-1}(d)$ (cf. [1, §5]). By working with the orbit space and using Corollary 1, we are able to prove

**Theorem 3.** For $n = 2k + 3$, $k = 1, 2, 3, \ldots$, and for any $d$, there exists a smooth, codimension-one foliation of the manifold $\Sigma^{n}(d)$.

Corollary 1 is due to Alberto Verjovsky whose conversation was of great value to me during the preparation of this work. Detailed proofs of the above theorems will appear elsewhere.

**Bibliography**


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