INFINITE RESISTIVE NETWORKS

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An infinite resistive network \( N \) consists of a connected, locally finite, oriented, infinite graph with branches \( B_1, B_2, \ldots \). To each branch \( B_i \) is associated a resistance \( r_i \geq 0 \). We are also given a voltage source, i.e., a finite 1-cochain \( E' \), and a current source, i.e., a finite 0-chain \( i \) satisfying \( \partial i = 0 \).

For each (real) 1-chain \( \mathbf{c} = \sum a_i B_i \), define \( \| \mathbf{c} \| \) by \( \| \mathbf{c} \|^2 = \sum a_i^2 r_i \).

**Theorem 1.** There exists a unique 1-chain \( \mathbf{I} \) such that:

(i) (Kirchhoff's current law). \( \partial \mathbf{I} + \mathbf{i} = 0 \).

(ii) (Kirchhoff's voltage law). For each finite cycle \( \mathbf{Z} \),

\[ \langle E', \mathbf{Z} \rangle = \langle R(\mathbf{I}), \mathbf{Z} \rangle, \]

where if \( \mathbf{I} = \sum a_i B_i \), then \( R(\mathbf{I}) \) denotes the 1-cochain \( R(\mathbf{I}) = \sum a_i B'_i \).

Of course \( (B'_i, B_i) = \delta_{ij} \).

(iii) (Finite power). \( \mathbf{I} \) is square summable, i.e., \( \| \mathbf{I} \| < \infty \).

(iv) There is a sequence \( \{ \mathbf{C}_j \} \) of finite 1-chains such that \( \partial \mathbf{C}_j + \mathbf{i} = 0 \)

and \( \| \mathbf{C}_j - \mathbf{I} \| \to 0 \).

**Theorem 2.** Let \( N_j \) be any sequence of subnetworks such that \( N_1 \subseteq N_2 \subseteq \cdots \) and \( \bigcup N_j = N \). Suppose \( N_1 \) is large enough to support the voltage source \( E' \) and the current source \( i \). Let \( \mathbf{I}_j \) be the unique current on \( N_j \) given by Theorem 1. Then \( \| \mathbf{I}_j - \mathbf{I} \| \to 0 \), where \( \mathbf{I} \) is the unique current on \( N \).

The proofs of these results, corollaries, and a full discussion will appear shortly in the IEEE Trans. Circuit Theory.

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