INFINITE RESISTIVE NETWORKS

BY HARLEY FLANDERS

Communicated by S. Smale, January 4, 1971

An infinite resistive network $N$ consists of a connected, locally finite, oriented, infinite graph with branches $B_1, B_2, \ldots$. To each branch $B_i$ is associated a resistance $r_i \geq 0$. We are also given a voltage source, i.e., a finite 1-cochain $E'$, and a current source, i.e., a finite 0-chain $i$ satisfying $\partial i = 0$.

For each (real) 1-chain $C = \sum a_i B_i$, define $\|C\|$ by $\|C\|^2 = \sum a_i^2 r_i$.

**Theorem 1.** There exists a unique 1-chain $I$ such that:

(i) (Kirchhoff's current law). $\partial I + i = 0$.

(ii) (Kirchhoff's voltage law). For each finite cycle $Z$, $\langle E', Z \rangle = \langle R(I), Z \rangle$, where if $I = \sum a_i B_i$, then $R(I)$ denotes the 1-cochain $R(I) = \sum a_i B'_i$. Of course $(B'_i, B_i) = \delta_{ij}$.

(iii) (Finite power). $I$ is square summable, i.e., $\|I\| < \infty$.

(iv) There is a sequence $\{C_j\}$ of finite 1-chains such that $\partial C_{j+1} = 0$ and $\|C_j - I\| \to 0$.

**Theorem 2.** Let $N_j$ be any sequence of subnetworks such that $N_1 \subset N_2 \subset \cdots$ and $\bigcup N_j = N$. Suppose $N_1$ is large enough to support the voltage source $E'$ and the current source $i$. Let $I_j$ be the unique current on $N_j$ given by Theorem 1. Then $\|I_j - I\| \to 0$, where $I$ is the unique current on $N$.

The proofs of these results, corollaries, and a full discussion will appear shortly in the IEEE Trans. Circuit Theory.

Tel Aviv University, Ramat Aviv, Tel Aviv, Israel

AMS 1970 subject classifications. Primary 94A20; Secondary 05C20, 46C05.

Key words and phrases. Infinite networks, resistive networks.