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THE FIRST BETTI NUMBERS OF CERTAIN LOCALLY TRIVIAL FIBRE SPACES

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It is well known that, for a compact, oriented, homogenous coset space $G/H$ arising from a compact, semisimple Lie group $G$, one has

$$b_1(G/H) \leq b_1(G) = 0.$$ 

In this note, we announce the following generalization of that result:

THEOREM 1. Let $\pi : E \to B$ be a locally trivial Riemannian fibre space, $E$ and $B$ compact, oriented Riemannian manifolds, with the fibres $F = \pi^{-1}(b)$ immersed in $E$ as minimal submanifolds. Then

$$b_1(B) \leq b_1(E).$$

We outline the proof.

From Hodge-deRham theory, we have $H^p(M, \mathbb{R}) \cong \mathcal{H}^p(M)$, the space of harmonic $p$-forms on the compact, oriented Riemannian manifold $M$. In [4], we show

THEOREM 2. Fix $p \geq 1$. If $\varphi : E \to B$ is a locally trivial fibre space mapping between compact, orientable Riemannian manifolds satisfying $\varphi^*\delta = \delta\varphi^*$ on all $p$-forms of the base manifold, then

$$b_p(B) \leq b_p(E).$$

We also show

THEOREM 3. Fix $p \geq 1$. Then $\varphi : E \to B$, a $C^3$ map between arbitrary compact oriented Riemannian manifolds, commutes with the codifferential.

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operator, $\delta$, on $p$-forms of $B$ if and only if
\begin{enumerate}
\item $\varphi$ is a $C^\infty$ locally trivial Riemannian fibre space mapping, and
\item $\bar{\delta}(\varphi_\bullet \wedge \cdots \wedge \varphi_\bullet)(p\text{-times}) = 0$,
\end{enumerate}
where $\varphi_\bullet \wedge \cdots \wedge \varphi_\bullet$ is the canonical tensor-valued $p$-form of type $(p,0)$ defined in [2] and [4] and $\bar{\delta}$ is the codifferential operator for such tensor-valued forms, dual to the exterior differentiation operator $\bar{\delta}$ defined in [4].

It is known that $\varphi$ is a harmonic mapping [1] if and only if $\bar{\Delta}(\varphi_\bullet) = 0$ if and only if $\bar{\delta}(\varphi_\bullet) = 0$. Moreover, a locally trivial fibre space mapping is harmonic if and only if its fibres are minimally immersed, because $\bar{\delta}\varphi_\bullet$ is essentially the trace of the O'Neill $T$-tensor for the fibre space mapping $\varphi$ and $T$ is the second fundamental form of the fibres when restricted to vertical vectors. Hence, Theorem 1 is outlined. Full proofs will appear elsewhere [4].

There are several immediate corollaries to Theorem 1. For instance,

**Corollary 1.** Let $\pi: P \to M$ be a Riemannian principal fibre bundle with compact Lie structure group $G$, and both $P$ and $M$ compact (e.g. bundle of frames of $M$). Then

$$b_1(M) \leq b_1(P).$$

Theorem 1 is a generalization of other similar results for the Laplacian operator found in [3].

**References**

4. ———, $\delta$-commuting maps and Betti numbers (submitted for publication).

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