less general and stick to the problems of uniform approximation, L^p-norms and Hilbert spaces stressing in the latter case the usefulness of reproducing kernels. Still they treat a more wide range of topics including Müntz' theorem, minimal extrapolation of Fourier transform and applications in complex analysis and feature in the introduction a good deal of the author's "philosophy" of approximation theory.

To be fair to the reader we have to admit that the book under review presents the state of art as of 1966 and that the present English edition was not updated. As of this writing, however, there appeared a great many papers having made a substantial contribution to the approximation theory in normed spaces. Obviously it is not the job to be done in this review to analyze the recent development of the subject but for the reader's convenience we attach to the references a short list of papers [7]–[10] which, as we feel, represent some of the recent trends and extend in some way or another, results presented by the useful book of Ivan Singer.

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Algebraic Number Theory by Serge Lang. Addison-Wesley, Reading, Mass., 1970, $14.95.

According to the foreword, *Algebraic Number Theory* is meant to supersede Lang's *Algebraic Numbers*. The earlier book could be split roughly into two parts: the first half was basic algebraic number theory, and the second was analytic. For this book, Lang inserted a section on class field theory in the middle. It must show something about terminology that Goldstein's book [2], which touches on many of the same topics, is called *Analytic Number Theory*.

The first three chapters are taken over from *Algebraic Numbers* with virtually no change. Chapter I goes through the basic facts about the algebraic integers in a number field, including remarks about prime ideals (and unique factorization), localization, integral closure, and a discussion of what happens to a prime ideal under a Galois extension. The main change from the earlier book is a more detailed discussion of how a prime splits (with an example). Chapter II, "Completions", deals with valuation theory (briefly) and the $\mathfrak{p}$-adic topologies; there is also a discussion of ramified and tamely ramified extensions. Lang has added proofs of some results on valuation theory for which he previously gave references. Chapter III is concerned with the different and discriminant; to his earlier account, Lang has added an example. (He also rearranged matters a little; one part of the old Chapter III is now in Chapter IV.)

After Chapter III, the books diverge somewhat more. Chapter IV, "Cyclotomic Fields", starts out with a discussion of roots of unity. It continues with an account of quadratic fields; the new book also gives a full proof of quadratic reciprocity. It continues with a short study of Gauss sums, and ends with a theorem of McKenzie's giving a relation in the ideal class group of $Q(\zeta)$, ζ a root of unity. Chapter V, "Parallelotopes", is primarily concerned with Minkowski's proofs of the finiteness of the class number and the Dirichlet unit theorem. It is virtually identical with the earlier Chapter V; the main change is that Lang adds a definition of the regulator.

From now on, I will skip around a bit, first taking care of the material also touched on in *Algebraic Numbers.* Chapter VII starts out by defining ideles and adeles; Lang adds some material which I will return to later. Chapter XIV, "Functional Equation, Tate's Thesis", is an almost exact copy of Chapter VII of *Algebraic Numbers*; it contains Tate's proof of the functional equation for L-series. (The copy is too nearly exact; Lang repeats the definition of the regulator that he gave in Chapter V.) But Lang adds something new in this book; Chapter XIII, "Functional Equation of the Zeta Function, Hecke's Proof" gives the classical account of the same material. (Lang restricts himself to zeta functions to keep the computations under control.) Chapter XIII also applies the zeta function to get estimates on the number, $j(x)$, of ideals of an algebraic number field with norm $<x$ (under some sort of assumption on the number of zeroes in the critical strip).

Chapter XV, "Density of Primes and Tauberian Theorem", proves

Ikehara's Tauberian Theorem and applies it to theorems on the density of primes in generalized arithmetic progressions. Lang has corrected the error in one of the theorems which he made in *Algebraic Numbers* (later editions had an erratum sheet with the correction), and has added some examples (included, in part, on the extra space on the errata sheet). Chapter XVI, "The Brauer-Siegel Theorem", is devoted to a proof of that theorem, and Chapter XVII, "Explicit Formulas", is a fuller account of Weil's paper [3]. These chapters are essentially the same as the last two chapters of *Algebraic Numbers*.

That leaves the new material in the book. It consists, primarily, of proofs of the main results of class field theory. Lang includes an interesting discussion of the history of the subject and of the various methods now known to prove the basic theorems (along with references). His basic approach is about as classical as possible; he gives Artin's original proof of reciprocity (with some simplifications, also due to Artin), and Weber's analytic proof of the second inequality. In fact, he phrases most of the discussion of the reciprocity law in the language of ideal class groups, bringing in ideles later.

Lang prepares this approach in Chapter VI, where he discusses the connection between ideal classes and idele classes. In Chapter VIII, he gives the basic material on L-series needed for the analytic proof of the second inequality, and proves it. He also proves Dirichlet's theorem on the density of primes in arithmetic progression (where the density is the so-called Dirichlet density), and Tchebotarev's theorem.

Chapter IX is rather technical, and Lang advises the reader to tackle it only after reading Chapter X. It consists primarily of proving the norm index equality in certain special cases (later absorbed into more general results). Chapter X takes care of the reciprocity law and the other main basic theorems of global class field theory, with one exception. That exception is the existence theorem, which is done in Chapter XI. Chapter XI also does local class field theory. Finally, Chapter XII discusses the Artin L-series and proves the result which originally motivated Artin's reciprocity law: that Artin L-series are ordinary L-series in the Abelian case.

There is a lot to praise in this book. Lang's treatment of class field theory is extremely clear and well written; he is concise, but simultaneously he keeps the reader well informed about where he is and where he is going. Much of the material in the last part of the book is not found in any other text (except *Algebraic Numbers*), and Lang presents it clearly and far more completely than in the original papers. Lang has the ability to expose material in a literal sense; that is, he can show the reader what is happening in a proof and why. When he is writing well, he writes very well.

But when he is not, all sorts of things can go wrong. Lang is often too brief in his treatment of a subject. I have felt for a while that many of his books seem much better to a person who knows the subject than to someone trying to learn from them. I suppose that it is because Lang often leaves out small details which students have trouble filling in. Another aspect of this is that too many results get dismissed as trivial. In fact, Lang cites one theorem as trivial twice in this book. (To be fair, I should add that he proves it both times.) Perhaps it is worth recalling the words of a brilliant reasoner (not a mathematician): "Every problem becomes very childish when once it is explained to you." (See **[1]**.)

Some parts of the book are sloppily written. Here are a few examples. Lang's proof of what he calls property QS1 (p. 85) contains an equation which needs to have a term removed. Formula (3) on p. 84 should read $|x(X, n)| = \sqrt{q}$ (not $\sqrt{n}$); Lang proves the correct formula and then asserts that he proved the wrong one. The proof of Theorem 1, p. 139, is wrong as it stands. Proposition 9 of Chapter II (pp. 49–50) was dismissed as trivial in *Algebraic Numbers*; in *Algebraic Number Theory*, he devotes some space to a proof and still leaves a rather large chunk to the reader. There are quite a few misprints, though generally they do not cause any trouble. It is a shame that these flaws are not corrected; they turn what could have been an excellent book into merely a good one. I realize that the author is responsible for the contents of his book, and that mathematics authors generally do their own proofreading; nevertheless, a better job of editing could have improved matters considerably.

One final matter. Just about every advanced mathematics textbook needs an index of symbols. This one certainly does. What would be ideal, in fact, would be a card giving a list of symbols, with brief definitions. Publishers provide lists of characters for copies of *War and Peace*; why not for *Algebraic Number Theory*?

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