BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

TWO CONSISTENCY RESULTS IN TOPOLOGY

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Communicated by Mary Ellen Rudin, March 24, 1972

The present authors proved in 1966 that for any cardinal α, if a Hausdorff space X is of cardinality $> 2^{2^\alpha}$ then it contains a discrete subspace of cardinality $> \alpha$. The following result shows that this cannot be improved (even for completely regular spaces).

THEOREM 1. Con(ZF) $\rightarrow$ Con(ZFC & $2^\alpha = \alpha^+$ & 2^{α^+} *is anything reasonable & there exists a zero-dimensional* T_2 *space* X *of cardinality* $2^{\alpha^+} = 2^{2^\alpha}$ *which is hereditary* α*-separable*).

(A space is hereditary α-separable iff every subspace in it has a dense subset of cardinality $\leqq \alpha$.)

COROLLARY. *It is consistent to assume that* 2^{ω_1} *is big and there exists a zero-dimensional* T_2 *space in which the number of all open sets is any cardinal* β *with* $\omega_1 \leqq \beta < 2^{\omega_1}$ *and* $\mathrm{cf}(\beta) \neq \omega$.

This relates to a problem raised by J. de Groot.

A space X is called α-Lindelöf if every open cover of it can be reduced to a subcover of cardinality $\leqq \alpha$. X is hereditary α-Lindelöf iff every subspace of X is α-Lindelöf.

THEOREM 2. *Let* α *be a given cardinal number. Then* Con(ZF) $\rightarrow$ Con(ZFC & GCH & $\exists$ *zero-dimensional* T_2 *space* X *so that*

(i) $|X| = \alpha^+$ *and* X *is hereditary* α*-Lindelöf*;

(ii) $Y \subset X$ *and* $|Y| = \alpha^+$ *imply that the weight of the subspace* Y *is* α^{++};

(iii) $Y \subset X$ *and* $|Y| \leqq \alpha$ *imply that* Y *is closed and discrete*).

COROLLARY. *For any given* α, *it is consistent to assume that there exists a zero-dimensional* T_2 *space of weight* α^{++} *in which no subspace has the weight* α^+.

AMS 1969 *subject classifications*. Primary 5440, 5423, 0430, 0415, 0250, 0263.
Key words and phrases. Hereditarily separable, zero dimensional, cardinality, Lindelöf, weight.

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