BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

A THULLEN TYPE EXTENSION THEOREM FOR POSITIVE HOLOMORPHIC VECTOR BUNDLES[1]

BY YUM-TONG SIU[2]

Communicated by S. S. Chern, April 3, 1972

We announce the following result.

THEOREM. *Suppose X is a complex manifold, A is an analytic subset of X of codimension $\geqq 1$, and G is an open subset of X which intersects every branch of A of codimension 1. Suppose V is a semipositive holomorphic vector bundle over $(X - A) \cup G$ (i.e. V carries a hermitian metric with positive semidefinite curvature form). Then the sheaf $\mathcal{O}(V)$ of germs of holomorphic sections of V can be extended uniquely to a reflexive coherent analytic sheaf over X.*

COROLLARY. *If* $\dim X = 2$, *then V can be extended uniquely to a holomorphic vector bundle over X.*

The special case where A has codimension $\geqq 2$ and V is a line bundle was proved by Shiffman [3], [4]. An alternative proof of Shiffman's line bundle result was given by Harvey [1] whose proof works also when A is an arbitrary closed subset of X with Hausdorff $(2 \dim X - 3)$-measure 0.

Our Corollary implies a theorem of Thullen [6, Satz 2], because, in a special case general enough to give the general case, the line bundle associated to the analytic subset of codimension 1 which is to be extended is semipositive.

The proof of our Theorem follows from Hörmander's L^2 estimates for the $\bar{\partial}$ operator [2] and the easy part of the usual sheaf-extension techniques (see e.g. [5] and related papers listed in the bibliography there). Let $\Delta_r = \{z \in C \mid |z| < r\}$ and $\Delta = \Delta_1$. We outline here the proof of our Theorem for the special case where $X = \Delta \times \Delta$, $A = \Delta \times \{0\}$, and $G = \Delta_{1/2} \times \Delta$.

Fix arbitrarily $\frac{1}{2} < r < 1$. Let $f_1, \ldots, f_k$ be holomorphic sections of V over $\Delta \times (\Delta - \{0\})$ generating $\mathcal{O}(V)$ there. Take arbitrarily $c \in \Delta - \{0\}$. Let $\rho = \rho(z_2)$ be a C^∞ function on $\Delta - \{0\}$ with compact support such that $\rho \equiv 1$ on a neighborhood of c. Since $(z_2 - c)^{-1}\bar{\partial}(\rho f_j) \mid \Delta_r \times (\Delta - \{0\})$ has

AMS 1969 *subject classifications*. Primary 3227, 3250; Secondary 3519.

Key words and phrases. Positive vector bundle, curvature form, coherent sheaf extension, differential operator estimates.

[1] Research partially supported by National Science Foundation Grant GP-27624.

[2] Sloan Fellow.

finite L^2-norm with respect to the given metric h of V, by Hörmander's method we can find a C^∞ section g_j of V over $\Delta_r \times (\Delta - \{0\})$ such that g_j has finite L^2-norm with respect to h and $\bar{\partial} g_j = (z_2 - c)^{-1}\bar{\partial}(\rho f_j)$. It is well known that a holomorphic function defined outside an analytic subset of codimension $\geqq 1$ can be extended across it if the function is locally L^2 at every point of the analytic subset. Hence $\rho f_j - (z_2 - c)g_j$ can be extended to a holomorphic section s_j of V over $(\Delta_r \times (\Delta - \{0\})) \cup G$. The sections $s_1, \ldots, s_k$ generate $\mathcal{O}(V)$ at $\Delta_r \times \{c\}$. Likewise we can find holomorphic sections of V over $(\Delta \times (\Delta_r - \{0\})) \cup (\Delta_{1/2} \times \Delta_r)$ generating $\mathcal{O}(V)$ at $\Delta_{1/2} \times \{0\}$. The Theorem for this case now follows from wellknown easy sheaf-extension techniques.

Theorems on extending semipositive holomorphic vector bundles across closed subsets with Hausdorff measure conditions can also be obtained.

Details will appear elsewhere.

References

1. F. R. Harvey, *A result on extending positive currents*, preprint 1971.
2. L. Hörmander, *L^2 estimates and existence theorems for the $\bar{\partial}$ operator*, Acta Math. **113** (1965), 89–152. MR **31** #3691.
3. B. Shiffman, *Extension of positive holomorphic line bundles*, Bull. Amer. Math. Soc. **77** (1971), 1091–1093.
4. ———, *Extension of positive line bundles and meromorphic maps*, Invent. Math. **15** (1972), 332–347.
5. Y.-T. Siu, *A Hartogs type extension theorem for coherent analytic sheaves*, Ann. of Math. (2) **93** (1971), 166–188.
6. P. Thullen, *Über die wesentlichen Singularitäten analytischer Funktionen und Flächen im Raume von n komplexen Veränderlichen*, Math. Ann. **111** (1935), 137–157.

Department of Mathematics, Yale University, New Haven, Connecticut 06520