Introduction. In this note we announce results concerning normal bundles, disc bundles, and Stiefel-Whitney classes in the topological category. Many of these results also hold in the piecewise linear (PL) category, but the dimensions should be restricted accordingly.

I would like to thank Professors R. F. Brown and R. Edwards for many helpful and encouraging discussions.

Normal bundles and disc bundles. Let $\text{TOP}_n$ be the semisimplicial (s.s.) group of topological origin-preserving homeomorphisms of $R^n$. Let $\text{TOP}_{n,k}$ be the s.s. group of topological homeomorphisms of $R^n = R^{n-k} \times R^k$ which are pointwise fixed on $R^k$.

In [9] Kirby and Siebenmann announced a strong stability theorem for $\text{TOP}/O$, i.e., if $n = 5$, the stability map

$$s_i: \pi_i(\text{TOP}_n, O_n) \to \pi_i(\text{TOP}_{n+1}, O_{n+1})$$

is an isomorphism for $i \leq n + 1$ and an epimorphism for $i = n + 2$, where $O_n$ is the s.s. $n$-dimensional orthogonal group. Using this result we deduce that

**Theorem 1.** $\pi_i(\text{TOP}_n, O_n) = 0$ for $i \leq n + 1$, $n \geq 6$, where $\text{TOP}_n(I)$ is the s.s. group of topological origin-preserving homeomorphisms of the unit disc in $R^n$.

An immediate corollary is

**Corollary 2.** Let $X$ have the homotopy type of a $k$-dimensional CW complex. Any $R^n$-bundle over $X$ contains a disc bundle if $n \geq k - 2$, $n \geq 6$. It is uniquely determined (up to isomorphism) if $n \geq k - 1$, $n \geq 6$.

In particular, every $n$-manifold, $n \geq 6$, has a tangent disc bundle.

Using the above stability result and results of Rourke and Sanderson ([12], [13]), we show that

**Theorem 3.** $\pi_i(\text{TOP}_{n,k}, \text{TOP}_{n-k}) = 0$ if $i \leq n - k + j$, $n - k \geq 5 + j$, $j = 0, 1, 2$.

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1 These results will appear in the author’s doctoral thesis at UCLA.

2 Research supported in part through an NSF Traineeship at UCLA.
Then, by the results of K. Millett [10], Browder [2], and Theorem 3, one can show the existence of disc bundles in Corollary 2 for $R^n$-bundles with nonzero cross section when $n = k - 3, n \geq 11$.

Using Theorem 3 and the immersion theorem of Lees or Gauld [7] we immediately obtain

**Corollary 4.** Let $M^n$ be a locally flat submanifold of $N^{n+q}$. If $q \geq 5 + i$, then $M$ has a normal open or microbundle in $N$ if $q \geq n - i - 1, i = 0, 1, 2$. If $q \geq n - i$ it is unique up to isotopy.

**Remark.** We obtain three more dimensions for disc bundles and open normal bundles than [13].

**Stiefel-Whitney classes.** Let $\xi = (E, E_0, p, B)$ be an $R^n$-bundle with zero-section, where $E_0 = E$-zero section, $p : E \to B$, and $B$ is compact having the homotopy type of an $m$-dimensional CW-complex. Let $w_i(\xi)$ denote the $i$th Stiefel-Whitney class of $\xi$. The top class of $\xi$, denoted $W_n(\xi)$, is given by $W_n(\xi) = \Phi^{-1}(U_\xi \cup U_\varnothing)$, where $U_\xi \in H^n(E, E_0; p^*\Gamma)$ is the Thom class, $\Gamma$ the local system of $B$ such that

$$\Gamma(b) = H_n[p^{-1}(b), p_0^{-1}(b); Z],$$

and $\Phi$ the Thom isomorphism.

Let $\xi(V_{n,k})$ be the bundle associated to $\xi$ with fiber $\text{TOP}_n/\text{TOP}_{n-k}$. Let $c_{n-k+1}(\xi)$ denote the primary obstruction to finding a cross section of $\xi(V_{n,k})$. Using Theorem 1, results of Akiba [1], and variations of classical techniques in the differentiable category we prove

**Theorem 5.** If $n \neq 4, 5, n \geq m - 2$, then $c_n(\xi) = 0$ iff $W_n(\xi) = 0$. If $n - k \geq 5, c_{n-k+1}(\xi)$ reduced mod 2 equals $w_{n-k+1}(\xi)$. Also $c_n(\xi) = \lambda W_n(\xi)$ where $\lambda = 0$ iff $W_n(\xi) = 0$.

Let $\xi(V_{n,k})$ be the bundle associated to $\xi$ with fiber $\text{TOP}_n/\text{TOP}_{n,k}$. Let $d_{n-k+1}(\xi)$ denote the primary obstruction to finding a cross section to $\xi(V_{n,k})$. Using a theorem of K. C. Millett [10] and Theorem 3, we show that

**Theorem 6.** If $n \neq 4, m + 5 \leq 2(n - k)$, then $d_n(\xi) = 0$ iff $W_n(\xi) = 0$. If $n - k \geq 5$, then $d_{n-k+1}(\xi)$ reduced mod 2 equals $w_{n-k+1}(\xi)$. Also $d_n(\xi) = \lambda W_n(\xi)$ where $\lambda = 0$ iff $W_n(\xi) = 0$.

These theorems provide a geometrical interpretation of the Stiefel-Whitney classes in the topological category similar to that in the differentiable category (see [11], [14]).

**Remark.** We can also prove results analogous to Theorems 4 and 5 in the PL category with no restrictions on $n - k$, by using results of Hirsch [8]. However, in Theorem 5 we must have $n \geq m$. 

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Let $M^n$ be a compact $n$-manifold with or without boundary. An arc-field on $M$ is a map $p: M \to M^1$ such that, for all $b \in M$, $p(b)(0) = b$ and $p(b)$ is a homeomorphism. Using results of R. F. Brown and E. Fadell \[3\], Hirsch \[8\], and Theorem 6 we make the following observation.

**Corollary 7.** If $M^n$ is a PL-manifold, or if $n \neq 4$, $M$ has an arc-field iff the Euler characteristic of $M$ is zero.

This was proven for all triangulated manifolds by Fadell \[5\].

**Remark.** In view of Theorems 5 and 6 the appropriate topological version of a tangent $k$-field should be a map $p: M \to M^{Rk}$ where, for all $b \in M$, $p(b)(0) = b$ and $p(b)$ is a locally flat homeomorphism. This answers a question posed by E. Fadell \[6\].

Details of proofs will appear elsewhere.

**References**


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