TOTALLY GEODESIC FIBRE MAPS

BY MU-CHOU LIU

Communicated by S. S. Chern, August 9, 1972

Let $M$ be a Riemannian manifold and $\Pi: TM \to M$ be its tangent bundle. There exist two kinds of naturally induced metrics on $TM$, the Sasaki metric and the pseudo-Riemannian metric ([3], [4]). If $TM$ is endowed with the Sasaki metric and $M$ is compact, we have shown that $TM$ is a complete Riemannian manifold which admits no negative curvature. In [4], Yano and Kobayashi determined the holonomy group of the pseudo-Riemannian connection on $TM$. A fibre map is said to be trivial if it collapses the whole fibre into a point.

Based on the results of Yano and Kobayashi, we prove the following

**Theorem 1.** Suppose $M$ and $N$ are Riemannian manifolds. If $F: TM \to TN$ is a totally geodesic fibre preserving map, then the induced map $f: M \to N$ is totally geodesic. If for some $u \in TM$, $\text{Ker} \ F^*_u$ contains a nonvertical vector, then $F$ is trivial.

By using the Morse theory and Cartan-Hadamard Theorem together with the above theorem, we prove the following

**Theorem 2.** Suppose $M$ is a Riemannian manifold, and suppose its Ricci curvature $K$ satisfies $K(X, X) \geq (n - 1)/c^2$ for every unit vector $X$ at every point of $M$, where $c$ is a positive constant. If there exists a geodesic of length greater than $\Pi c$, and if $N$ is a complete Riemannian manifold of negative curvature, then any fibre preserving totally geodesic map $F: TM \to TN$ is trivial.

**Corollary.** If $f: M \to N$ is a map such that the tangent map $f^*: TM \to TN$ is totally geodesic, then $f$ is a constant map.

A direct consequence of Theorem 2 is the following:

**Theorem 3.** Suppose $M$ is a compact Riemannian manifold with everywhere positive definite Ricci tensor. If $N$ is a Riemannian manifold of negative curvature, then any fibre preserving totally geodesic map $F: TM \to TN$ is trivial.

The proofs of these results will appear in [2].

---


Key words and phrases. Totally geodesic fibre map, induced pseudo-Riemannian connection, holonomy group, Ricci curvature.

¹ Work partially supported by NSF grant 24917.
REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, CHICAGO, ILLINOIS 60680