BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 2, March 1973

# MINIMUM COVERS FOR ARCS OF CONSTANT LENGTH

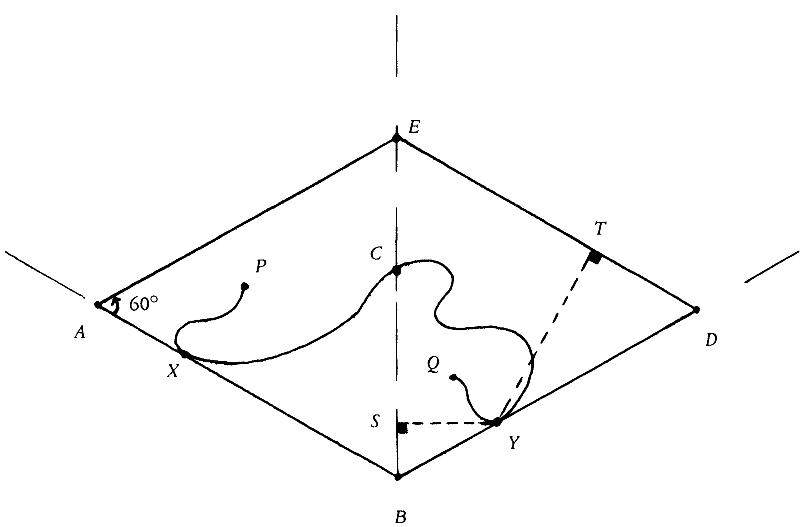
BY GEORGE POOLE AND JOHN GERRIETS

Communicated by Mary Ellen Rudin, September 22, 1972

Recently Gerriets [**1**] showed that a certain convex closed region with area less than $0.3214L^2$ covers any arc of length $L$. This is an improvement to Wetzel's results [**3**] on the famous and elusive "Worm Problem" of Leo Moser [**2**]: What is the (convex) region of smallest area which will accommodate every arc of length $L$? Wetzel showed that a certain truncated sector with area less than $0.34423L^2$ covers all arcs of length $L$. By slightly modifying the region considered by Gerriets, we obtain a region with area less than $0.2887L^2$ which covers any arc of length $L$.

THEOREM. *The closed region whose boundary is a rhombus with major diagonal $L$ and minor diagonal $L/3^{1/2}$ covers any arc of length $L$.*

Herein we give a sketch of the proof. Details and other results will appear elsewhere. Let $PQ$ denote an arc of length $L$ and with center $C$ whose two subarcs are $PC$ and $CQ$. "Slide" the arc $PQ$ along $BE$ toward $B$ so that $C$ is always incident with $BE$ and $PQ$ becomes "tangent" to $AB$ or $BD$ (see the figure below) at the points $X$ or $Y$. It is possible that all such




orientations of $PQ$ by rotation allow only one of the arcs $PC$ or $CQ$ to be

*AMS (MOS) subject classifications* (1970). Primary 52A45, 52A40.
*Key words and phrases.* Arcs, convex regions.

tangent to the angle $ABD$ with all other points of the arc $PQ$ lying on or above the angle. Assume for the present, however, that there are two tangent points $X$, $Y$ on $PC$, $CQ$ which lie on the segments $AB$ and $BD$, respectively. Construct the segments $SY$ and $YT$ perpendicular to $BE$ and $DE$, respectively (the case when a point between $C$ and $Y$ meets $DE$ is handled in a similar way to the case under discussion). If $CQ$ agrees with the segments $SY$ and $YT$, then the length of $CQ$ is exactly $L/2$ and, hence, is covered by the region $R$ described in the Theorem. If $CQ$ does not agree with $SY$ and $YT$, then in order for $CQ$ to get to the boundary, its length must exceed the length of $SYT$ (which is $L/2$), an impossibility. So $CQ$ is covered by $R$ and, similarly, $PC$ is also covered. Symmetry of $R$ dispenses with the case that $PQ$ has only one subarc tangent to $ABD$.

It can be shown that the region $R$ can be truncated to obtain a region with area less than $0.2861L^2$ which covers any arc of length $L$.

The authors wish to thank Professor John E. Wetzel for sharing the results in [3] prior to publication.

## References

1. John Gerriets and George Poole, *Covers accommodating curves of constant length* (submitted).

2. Leo Moser, *Poorly formulated unsolved problems of combinatorial geometry* (mimeographed).

3. John E. Wetzel, *Sectorial covers for curves of constant length*, Canad. Math. Bull. (to appear).

Department of Mathematics, Kansas State Teachers College, Emporia, Kansas 66801