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SURFACES WITH A PARALLEL ISOPERIMETRIC SECTION

BY BANG-YEN CHEN

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This announcement is a continuation of Chen [**1**] (also, Yau [**3**]). We shall present additional theorems relating surfaces in a space form with a parallel normal section.

Let M be a surface in an m-dimensional Riemannian manifold R^m with the induced normal connection D. For a unit normal section ξ on M (that is, a unit normal vector field of M in R^m), let A_ξ be the second fundamental tensor with respect to ξ; if we have $D\xi = 0$ identically, then ξ is called a *parallel section*; if the trace of A_ξ is constant (respectively, zero), then ξ is called an *isoperimetric section* (respectively, *minimal section*) on M; if the determinant of A_ξ is nowhere zero, then ξ is called a *nondegenerate section*; if A_ξ vanishes identically, then ξ is called a *geodesic section*; and if A_ξ is not proportional to the identity transformation everywhere, then ξ is called a *umbilical-free section.*

THEOREM 1. *Let M be a closed surface in an m-dimensional Riemannian manifold R^m of constant sectional curvature such that the Gaussian curvature of M does not change its sign. If there exists a parallel umbilical-free isoperimetric section on M, then M is flat.*

THEOREM 2. *Let M be a closed surface of a 4-dimensional Riemannian manifold R^4 of constant sectional curvature $c \leqq 0$ such that the Gaussian curvature of M does not change its sign. If there exists a parallel nondegenerate minimal section on M, then the mean curvature vector of M is parallel.*

THEOREM 3. *Let M be a surface in an m-dimensional simply-connected complete Riemannian manifold R^m of constant sectional curvature c such that the Gaussian curvature of M is constant. If there exists a parallel isoperimetric section on M, then either M is contained in a (small or great) hypersphere of R^m or M is flat.*

THEOREM 4. *Let M be a surface in a 4-dimensional simply-connected complete Riemannian manifold R^4 of constant sectional curvature $c \leqq 0$ such that the Gaussian curvature of M is constant. If there exists a parallel minimal section on M, then either M is contained in a great hypersphere of R^4 or the mean curvature vector H of M is parallel and M is flat.*

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The proofs of these four theorems use the theory of analytic functions.

From Theorems 2 and 4 we have immediately the following characterization theorems of standard flat tori in euclidean 4-space.

COROLLARY 1. *Let M be a closed surface in a euclidean 4-space such that the Gaussian curvature of M does not change its sign. Then M is the product surface of two plane circles if and only if there exists a parallel nondegenerate minimal section on M (Chen* [**2**]).

COROLLARY 2. *Let M be a surface in a euclidean 4-space such that the Gaussian curvature of M is constant. Then M is an open piece of a product surface of two plane circles if and only if there exists a nongeodesic parallel minimal section on M.*

The detailed proofs of these results will be published in the author's forthcoming book *Geometry of submanifolds* published by Marcel-Dekker Inc., New York.

REMARKS. 1. If R^m is euclidean, Theorem 1 was also obtained by B. Wegner.

2. It was pointed out by H. B. Lawson that Corollary 2 of [**1**] should add the following assumption "M is not minimal surface of any hypersphere of E^m". The author would like to express his thanks to Professor Lawson.

REFERENCES

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2. ———, *A characterization of standard flat tori*, Proc. Amer. Math. Soc. **37** (1973), 564–567.

3. S.-T. Yau, *Submanifolds with constant mean curvature* (to appear).

DEPARTMENT OF MATHEMATICS, MICHIGAN STATE UNIVERSITY, EAST LANSING, MICHIGAN 48823