We announce here some results of a paper to appear elsewhere [1].

Let a torus $T$ act continuously on a topological space $X$. Let $X \to X_T \to B_T$ be the fibre bundle with fibre $X$ associated (by means of the action of $T$ on $X$) to the universal principal $T$ bundle $T \to E_T \to B_T$. We define the equivariant cohomology ring $H^*_T(X) = H^*(X_T)$ where $H^*$ denotes Čech cohomology with rational coefficients. When $Y$ is an invariant subspace of $X$, we define $H^*_T(X, Y) = H^*(X_T, Y_T)$. Then $R = H^*(B_T)$ is a polynomial ring and $H^*_T(X, Y)$ is a module over $R$ by means of $\pi^*$.

For each subtorus $L$ of $T$ let $PL$ be the kernel of $H^*(B_T) \to H^*(B_L)$. Let $X^L = F(L, X)$ be the set of points fixed by $L$. We will assume that $X$ is compact. Given a closed invariant subspace $Y \subset X$ and an element $x \in H^*_T(Y)$, we define

$I_x = \{ a \in R \mid ax \text{ lies in the image of } H^*_T(X) \to H^*_T(Y) \}$, and

$I^L_x = \{ a \in R \mid ax \text{ lies in the image of } H^*_T(X^L \cup Y) \to H^*_T(Y) \}$.

When $L \subseteq K$ are subtori, $I_x \subseteq I^L_x \subseteq I^K_x$. We say that $K$ belongs to $x$ if $K$ is maximal with respect to the property $I^K_x \neq R$.

1. Theorem. The isolated primary components of the ideal $I_x$ are the ideals $I^K_x$ where $K$ belongs to $x$. The radical of $I^K_x$ is $PK$, hence $\sqrt{I_x} = \bigcap PK$ where $K$ ranges over the subtori belonging to $x$.

2. Corollary. If $I_x$ is principal, the subtori belonging to $x$ are all of corank 1 and $I_x = \bigcap I^K_x$ where $K$ ranges over the subtori belonging to $x$. For each such $K$, $I^K_x = (\omega^d)$ where $d \geq 1$ and $\omega \in H^2(B_T)$ generates $PK$.

Assume that the fixed point set $F$ of the $T$ action on $X$ is not connected. Let $F = F^1 + \cdots + F^s$ be the connected components of the fixed point set, $s \geq 2$. We say that a subtorus $L$ connects $F^1$ and $F^2$ if they lie in the same component of $X^L$. We assume that $\dim H^*(X)$ is finite.

3. Theorem. Let $N \subset H^*_T(X)$ be the ideal generated by odd degree and $R$ torsion elements. Assume that $H^*_T(X)/N$ is generated by $k$ elements as an $R$ algebra. Then for every maximal subtorus $K$ connecting $F^1$ and $F^2$, rank $K \geq \text{rank } T - k$.

4. Remark. This generalizes a result of Hsiang \[3\] that \( F \) is connected whenever \( H^*_\mathbb{Q}(X) \) is generated as an \( R \) algebra by odd degree and \( R \) torsion elements.

The following proposition is a technical result related to a theorem of Golber \[2\].

5. Proposition. Assume that \( \dim H^*(X) = \dim H^*(F) < \infty \). Let \( S = \{ x \in X \mid \text{rank } T_x \geq \text{rank } T - 1 \} \). Then the homomorphism \( H^*_\mathbb{Q}(X, F) \to H^*_\mathbb{Q}(S, F) \) is injective.

We use the notation \( X \sim Y \) to indicate that there is an isomorphism of rational cohomology rings \( H^*(X) = H^*(Y) \). When \( X \sim S^{k_1} \times \cdots \times S^{k_n} \) where the \( k_i \) are odd integers, we define \( e(X) \) to be the second symmetric polynomial \( \sum_{i<j} (k_i + 1)(k_j + 1) \). If \( \dim H^*(X) = \dim H^*(F) \), we know that \( X^L \sim S^{d_1} \times \cdots \times S^{d_t} \) where the \( d_i \) are odd integers, for every subtorus \( L \) of \( T \) \[3\]. Hence \( e(X^L) \) is defined. Further we define \( g(X) = e(X) - e(F) - \sum L [e(X^L) - e(F)] \) where \( L \) ranges over the corank 1 subtori. For each subtorus \( H \) of corank 2, we define \( g(X^H) \) by using the induced \( T/H \) action on \( X^H \).

6. Proposition. \( g(X) = \sum_H g(X^H) \) where \( H \) ranges over the corank 2 subtori.

7. Remark. Golber \[2\] has proved that \( g(X) = \sum g(X^H) \) when \( X \sim S^{k_1} \times S^{k_2} \) where the \( k_i \) are odd, and \( F = \emptyset \).

When \( X \) is a compact rational cohomology manifold and \( F = F^1 + \cdots + F^s \) are the components of the fixed point set, let \( f_i \) be a generator of the top dimensional cohomology group of \( F^i \). After including \( f_i \in H^*(F^i) \subseteq H^*(F) \subseteq H^*_\mathbb{Q}(F) \), we can define the ideal \( I_{f_i} \). The following result was conjectured by Hsiang. It is a kind of splitting principle or Schur lemma for torus actions.

8. Theorem. The ideal \( I_{f_i} \) is principal with a generator of degree \( \dim X - \dim F^i \). This generator splits as a product of linear factors in \( R \) corresponding to the subtori belonging to \( f_i \).

Here \( n = \dim X \) means that \( H^n(X) \) is the top dimensional nonzero cohomology group of \( X \). We do an explicit computation of \( I_{f_i} \), when \( X \sim \text{quaternionic projective } n \text{ space} \[1\].

9. Remark. Theorem 8 holds for torus actions on Poincaré duality spaces. It also holds for actions of \( p \)-tori on Poincaré duality spaces over \( \mathbb{Z}_p \). The Borel formula (see \[3\]) also holds for such actions \[5\].

Theorem 8 yields the following result of Hsiang and Su \[4\].

10. Theorem. When \( X \) is a compact rational cohomology manifold and \( X \sim \mathbb{Q}P^n \), quaternionic projective \( n \) space, and a torus of rank \( \geq 2 \) acts
effectively on $X$, the fixed point set has at most one component $\sim Q^p_k$ with $k \geq 1$.

The results announced here also hold for actions of $p$-tori using $\mathbb{Z}_p$ cohomology.

REFERENCES