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ON THE DIOPHANTINE EQUATION $Y^2 + K = X^5$

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In this paper we shall discuss the integral solutions of the diophantine equation $Y^2+K=X^5$, where K is a square-free positive integer. We shall prove the following:

THEOREM. *If the class number h of the quadratic field* $Q(\sqrt{-K})$ *is not divisible by* 5, *and if* $K\neq 8L-1$, *then the equation* $Y^2+K=X^5$ *has no nonzero integral solutions with the exceptions of* $K=19$, 341.

In these cases the solutions will be as follows:

$$(22434)^2 + 19 = (55)^5,$$
$$(2759646)^2 + 341 = (377)^5. \quad \square$$

The ideal equation $[Y+\sqrt{-K}]\cdot[Y-\sqrt{-K}]=X^5$ leads to finitely many equations [see e.g. [**3**]] of the form $f(A, B)=m$, where f is a homogeneous polynomial of degree 5.

The case $Y+\sqrt{-K}=\omega^5$, where ω is an integer in $Q(\sqrt{-K})$ is reduced to solving $Y^2=20X^4+1$. This was discussed by W. Ljunggren [**2**] and J. H. E. Cohn [**1**].

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