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ADDENDUM TO: "ON EXTENSIONS OF FUNDAMENTAL GROUPS OF SURFACES AND RELATED GROUPS"

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Copying the methods of J. Nielsen **[1]** Theorem 1 of **[2]** can be proved, i.e. that a finite torsionfree extension of the fundamental group of a surface is isomorphic to the fundamental group of a surface. Indeed, the following slightly more general theorem can be proved, but it is considerably weaker than Theorem 1′ of **[2]**.

THEOREM. *Let $\mathfrak{F}$ be the fundamental group of a surface S and let $\mathfrak{F}$ be finitely generated. Let $\mathfrak{G}$ be a group which contains $\mathfrak{F}$ as a normal subgroup of finite index and which has the following properties:*

(i) *For each $g \in \mathfrak{G}$ the automorphism of $\mathfrak{F}$ defined by $x \mapsto g^{-1}xg$ is induced by a homeomorphism of S.*

(ii) *If $g \in \mathfrak{G}$ and $g^{-1}xg = x$ holds for all $x \in \mathfrak{F}$, then $g \in \mathfrak{F}$.*

(iii) *If $x^a = y^b = (xy)^c = 1$ holds for $x, y \in \mathfrak{G}$ and $a, b, c \geqq 2$, then x, y generate a cyclic subgroup of $\mathfrak{G}$.*

Then $\mathfrak{G}$ is isomorphic to a finitely generated discontinuous group of motions of the hyperbolic or euclidean plane.

I shall briefly sketch a proof of the Theorem which generalizes **[1]**. Let S be an orientable surface with finite genus and a finite number of holes and without boundary. We consider S as a Riemann surface. If the universal cover is holomorphically equivalent to the euclidean plane, everything can be proved in a similar way as in [**2**, Theorem 3]. Therefore we may assume that the universal cover is the hyperbolic plane $\boldsymbol{H}$ which we represent by the unit disk $\{z \in \boldsymbol{C} \mid |z| < 1\}$ and the Poincaré model. The fundamental group of S acts on $\boldsymbol{H}$ as a group $\mathfrak{F}$ of conformal transformations. We may assume that $\mathfrak{F}$ contains only hyperbolic transformations except the identity. Then the methods of **[1]** can be applied: Each cyclic subgroup of $\mathfrak{F}$ consists of motions with the same axis, and a maximal cyclic subgroup contains all elements preserving an axis. Therefore each automorphism of $\mathfrak{F}$ induces a permutation of the axes of $\mathfrak{F}$ and

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of their base points, which lie on $\partial \boldsymbol{H}=\{z \in \boldsymbol{C} \mid |z|=1\}$. If the automorphism is induced by a homeomorphism of S (which corresponds to a $\mathfrak{F}$-invariant homeomorphism of $\boldsymbol{H}$) the mapping of the set of base points can be extended to a homeomorphism of $\partial \boldsymbol{H}$ (this extends the homeomorphism of $\boldsymbol{H}$ to a homeomorphism of the closed unit disk). So $\mathfrak{G}$ defines a group of permutations of the axes of $\mathfrak{F}$. For $g \in \mathfrak{G}$ and an axis A, denote by gA the image axis. An axis A is *simple*, if $gA \cap A \neq \varnothing$ for $g \in \mathfrak{G}$ implies $gA=A$. We may restrict ourselves to the case where the elements of $\mathfrak{G}$ are induced by orientation preserving homeomorphisms of S. Now we can repeat the arguments of [**1**, pp. 51–78], in this more general situation and we obtain

LEMMA 1. *If $\mathfrak{G}$ admits a simple axis, then $\mathfrak{G}$ is isomorphic to a finitely generated discontinuous group of motions of the hyperbolic plane* $\boldsymbol{H}$.

The criterion for the existence of a simple axis is the same as that in Nielsen [**1**, pp. 78–94]:

LEMMA 2. *$\mathfrak{G}$ admits a simple axis, if* (iii) *holds.*

REMARK. If the group $\mathfrak{G}$ contains elements x, y with $x^a=y^b=(xy)^c=1$, $a, b, c \geqq 2$, which generate a noncyclic subgroup $\mathfrak{U}$, then it must be proved that the above relations are defining relations for $\mathfrak{U}$ (which I could not obtain in all cases) and that the index of $\mathfrak{U}$ in $\mathfrak{G}$ is "small". The conclusion in [**1**, pp. 99, lines 10–21], does not seem correct to me.

REFERENCES

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