ON A CLASS OF MINIMAL CONES IN \( \mathbb{R}^n \)

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Communicated by S. S. Chern, October 12, 1973

1. Introduction. In what follows let \( S^p(\rho) = \{ x \in \mathbb{R}^{p+1} \mid |x| = \rho \} \) and \( S_{p,q} = S^p((p/((p+q)^{1/2})) \times S^q((q/((p+q)^{1/2})) \subset S^{p+q+1}(1) \). Let \( M \) be a codimension 1, closed minimal submanifold of \( S^{n+1}(1) \) and \( C(M) = \{ tx \mid 0 < t < 1, x \in M \} \).

It is well known that \( C(M) \) is a minimal submanifold of \( \mathbb{R}^{n+2} \). An important question is whether \( C(M) \) minimizes area in \( \mathbb{R}^{n+2} \) with respect to its boundary \( M \). With respect to this question the following results are known:

(a) When \( n \leq 5 \), Simons [4] has given a negative answer.

(b) When \( M = S_{p,p} \), \( p \geq 3 \), Bombieri-De Giorgi-Giusti [1] have given an affirmative answer.

(c) When \( M = S_{p,q} \) and either \( p+q \geq 7 \) or \( p=q=3 \) Lawson [2], using a different approach from Bombieri-De Giorgi-Giusti, has given an affirmative answer.

(d) Lawson has also proved that when \( n=6 \) or \( n=7 \) the set of minimal cones \( C(M) \), that minimize area in \( \mathbb{R}^{n+2} \) with respect to their boundary \( M \), is finite up to diffeomorphisms.

In this note we answer the question when \( M = S_{p,q} \) with \( p+q=6 \).

2. Results. Using techniques related to those of Bombieri-De Giorgi-Giusti, we were able to prove in [3] the following two theorems:

**Theorem 1.** If \( p+q = n \) and either

(a) \( n \geq 7 \) or

(b) \( n=6 \) with \( |p-q| \leq 4 \),

then the cone \( C(S_{p,q}) \) minimizes area in \( \mathbb{R}^{n+2} \) with respect to its boundary \( S_{p,q} \).

**Theorem 2.** \( C(S_{1,5}) \) and \( C(S_{6,1}) \) do not minimize area in \( \mathbb{R}^8 \) with respect to their respective boundaries \( S_{1,5} \) and \( S_{6,1} \).


\textit{Key words and phrases.} Area minimizing current, rectifiable current, oriented tangent cone, plateau problem.

\(^{1}\) Partially supported by Conselho Nacional de Pesquisas (Brazil).
Now let $V$ be a $C^2$ vector field in $R^{n+2}$, having compact support not containing $S^{n+1}(1)$ and let $\{\phi_t\}$ be its 1-parameter group of diffeomorphisms. We say that $C(M)$ is stable if for any such vector field there is $\varepsilon > 0$ such that

$$\text{Area of } \phi_t(C(M)) \geq \text{Area of } C(M) \text{ when } |t| < \varepsilon.$$ 

By an argument similar to one in Simons [4] one may prove that $C(S_{1,8})$ and $C(S_{5,1})$ are stable.

So we have the following:

**Theorem 3.** Although $C(S_{1,8})$ and $C(S_{5,1})$ are stable they do not minimize area in $R^8$ with respect to their respective boundaries $S_{1,8}$ and $S_{5,1}$.

**Bibliography**


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