Let $C^n$ denote the space of $n$ complex variables $z=(z_1, \ldots, z_n)$ with Euclidean norm $\|z\|$. The open unit ball $\{z \in C^n : \|z\|<1\}$ is denoted by $B^n$. We consider holomorphic functions $f(z)=(f_1(z), \ldots, f_n(z)), z \in B^n$, from $B^n$ into $C^n$. The second derivative of such a function is a symmetric bilinear operator, $D^2f(z)(\cdot, \cdot)$ on $C^n \times C^n$, and $D^2f(z)(z, \cdot)$ is the linear operator obtained by restricting $D^2f(z)$ to $z \times C^n$, with matrix representation

$$D^2f(z)(z, \cdot) = \left( \sum_{m=1}^{n} \frac{\partial^2 f_k(z)}{\partial z_j \partial z_m} z_m \right), \quad 1 \leq j, k \leq n.$$ 

A locally biholomorphic mapping $f(z)$ from a domain $G \subset C^n$ into $C^n$ is said to be $K$-quasiconformal in $G$ if $\|Df(z)\|^n \leq K|\det Df(z)|, z \in G$, where $\|\|$ denotes the standard operator norm $\|A\| = \sup\{\|Aw\| : \|w\| \leq 1\}, A \in \mathcal{L}(C^n)$.

The purpose of this note is to announce the following $n$-dimensional ($n \geq 1$) generalizations of one-dimensional results due to J. Becker [1].

**Theorem.** Let $f(z)$ with $Df(0)=I$ be locally biholomorphic in $B^n$ and satisfy

$$(1) \quad (1 - \|z\|^2) \|Df(z)^{-1}D^2f(z)(z, \cdot)\| \leq c, \quad z \in B^n.$$ 

If $c \leq 1$ then $f$ is univalent in $B^n$ and

$$\|z\|/(1 + c \|z\|^2) \leq \|f(z)\| \leq \|z\|/(1 - c \|z\|^2), \quad z \in B^n.$$ 

If $f$ is $K$-quasiconformal in $B^n$ and $c<1$ then $f$ is univalent and continuous in the closed ball, $\overline{B^n}$, and $f$ can be extended to a quasiconformal homeomorphism of $R^{2n}$ onto $R^{2n}$.

For $n=1$, (1) is $|zf''(z)f'(z)| \leq c/(1-|z|^2)$, the local biholomorphy implies $f$ is 1-quasiconformal in $B^1$, and our theorem coincides with...
Becker's results. The quasiconformal extension of \( f \) is not holomorphic on all of \( C^n \), but viewed as a mapping of \( \mathbb{R}^{2n} \) to \( \mathbb{R}^{2n} \), it is ACL, differentiable a.e., and the dilatation is uniformly bounded a.e. (cf. [5, p. 115]). By arguments similar to Becker's, we derive these results from our \( n \)-dimensional generalization [3] of Pommerenke's theory of subordination chains [4], and from the following lemma.

**Lemma.** Let \( f(z) \) be locally biholomorphic and \( K \)-quasiconformal in \( B^n \). If \( f \) satisfies
\[
(1) \quad \| Df(z) \| = O(1/(1 - \|z\|^c)), \quad z \in B^n,
\]
and \( f \) has a continuous extension to \( \tilde{B}^n \) that satisfies a Lipschitz condition
\[
(2) \quad \| f(z) - f(w) \| \leq M \| z - w \|^{1-c}, \quad z, w \in \tilde{B}^n.
\]

The proof of (2) is fairly elementary, and does not require the use of subordination chains. The proof that (2) implies (3) depends upon \( n \)-dimensional versions of classical theorems of Hardy and Littlewood [2, pp. 361–363].

Complete proofs of our results and details of the theory of \( n \)-dimensional subordination chains will be submitted for publication elsewhere [3].

**REFERENCES**


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