BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

THREE STRUCTURE THEOREMS IN SEVERAL COMPLEX VARIABLES

BY REESE HARVEY

The purpose of this article is to describe three recent structure theorems in the theory of several complex variables and to point out a few of the many applications of these three theorems. In the first section we discuss a characterization of those currents (defined on an open subset of $\boldsymbol{C}^n$) which correspond to integration over complex subvarieties. The second section is concerned with the structure of positive, d-closed currents. Finally, in the third section, a characterization of boundaries of complex subvarieties of $\boldsymbol{C}^n$ is discussed. A common thread in the techniques of proof involves "potential theory" for several complex variables.

1. **Recognizing currents that correspond to integration over complex subvarieties.** Suppose V is a complex subvariety of an open set in $\boldsymbol{C}^n$ with each irreducible component of V of dimension k. It is sometimes useful to consider, instead of the point set V, the linear functional "integration over V", which we denote by $[V]$. More precisely, for each compactly supported smooth form φ of degree $2k$, define $[V](\varphi)$ by integrating φ over the manifold points of V. A basic fact about complex subvarieties is that in a neighborhood of a singular point the $2k$-volume of the manifold points is finite (see **[4]**, **[16]**, or **[24]**). Therefore $[V](\varphi)$ is locally estimated by a constant times the supremum of the coefficients of the form φ. This implies that $[V]$ is a current (of real dimension $2k$ or (real) degree $2n-2k$). In fact, this estimate implies that the current $[V]$ viewed as a differential form with distribution coefficients actually has measures for coefficients.

There are several ways of recognizing which currents are of the form $[V]$ where V is a complex subvariety. The most elementary result of this kind says that if V is a real $2k$ dimensional submanifold of $\boldsymbol{C}^n \cong \boldsymbol{R}^{2n}$ and at each point of V the tangent space to V, considered as a real linear subspace of $\boldsymbol{R}^{2n} \cong \boldsymbol{C}^n$, is in fact a complex linear subspace, then V is a complex submanifold. It will shed light on later results to reinterpret this elementary result as follows. Suppose u is a current of degree $2(n-k)$ (dimension $2k$)

An expanded version of an invited address delivered to the Society at the 78th Summer Meeting in Missoula, Montana, 1973; received by the editors December 14, 1973. Research partially supported by a Sloan Fellowship and NSF GP-19011.

AMS (MOS) subject classifications (1970). Primary 32-02, 32C25; Secondary 49F20, 37F05.

corresponding to integration over a smooth $2k$ dimensional oriented submanifold V of an open set in $\mathbf{R}^{2n}\cong\mathbf{C}^n$. Then V is a complex submanifold if and only if u is of bidegree $n-k$, $n-k$ (or bidimension k, k); where a current u is said to be of *bidegree* $n-k$, $n-k$ (*bidimension* k, k) if $u(\varphi)=0$ for all $\varphi=f\,dz^I\wedge d\bar{z}^J=f\,dz_{i_1}\wedge\cdots\wedge dz_{i_p}\wedge d\bar{z}_{j_1}\wedge\cdots\wedge d\bar{z}_{j_q}$ except when $p=q=k$ (where f is a smooth compactly supported function). The proof of the above fact easily reduces to the trivial case where V is a real linear subspace of $\mathbf{R}^{2n}\cong\mathbf{C}^n$.

Another way of recognizing currents corresponding to integration over subvarieties is provided by a special case of a real-variable result of Federer (see [**4**, 4.1.15 p. 373]). This result implies that if a current of real dimension $2k$ defined on an open subset of $\mathbf{C}^n$ is, closed under exterior differentiation, supported on an irreducible complex subvariety V, and of a special type called locally flat, then the current is a constant multiple of $[V]$ (see King [**14**, Proposition 3.1.3]). For an example of an application of this result suppose that f is a holomorphic function. Then $\log|f|$ is plurisubharmonic (and hence locally integrable) and in fact pluriharmonic outside $V=\{z:f(z)=0\}$ (see [**12**] or [**18**] for a discussion of plurisubharmonic functions). A *pluriharmonic function* is a function which is annihilated by all of the operators $\partial^2/\partial z_i\,\partial\bar{z}_j$. Therefore $(i/\pi)\,\partial\bar{\partial}\log|f|$ is a current supported on V. Using the result mentioned above it is easy to see that:

> $(i/\pi)\,\partial\bar{\partial}\log|f|$ is the current $\sum m_j[V_j]$ where $\{V_j\}$ is the family of irreducible components of V and each $m_j\in\mathbf{Z}^+$ is the multiplicity of f vanishing on V_j.

This formula is of fundamental importance in several complex variables (for example in Nevanlinna theory and residue theory).

Next we examine special properties of currents of the form $[V]$ (where V is a complex subvariety), or more generally of integral linear combinations. Suppose $\{V_j\}$ is a sequence of irreducible subvarieties of dimension k, which satisfy the condition that only a finite number of them intersect any given compact set, and that $\{m_j\}$ is a sequence of integers. Then the sum $\sum m_j[V_j]$ is called a *holomorphic k-chain*. (If $k=n-1$ this notion is equivalent to the classical notion of a divisor.) If, in addition, each m_j is positive then $\sum m_j[V_j]$ is called a *positive holomorphic k-chain*.

Now we list some of the properties of a holomorphic k-chain $u=\sum m_j[V_j]$.

(1) u is of bidimension k, k (bidegree $n-k$, $n-k$).

(2) u is d-closed.

Here d denotes exterior differentiation. This condition can be interpreted geometrically as saying that each V_j has no boundary, and is rigorously deduced from the fact that du must be supported in the singular points of the V_j (i.e., a set of real dimension $\leqq 2k-2$) which is too small to support a $2k-1$ dimensional boundary (see Federer [**4**] for the details).

(3) u has measure coefficients and the $2k$-density $\Theta_{2k}(u, z)$ is a positive integer at each point z in the support of u.

Here $\Theta_{2k}(u, z) \equiv \lim_{r\to 0^+} M_{B(z,r)}(u)/c_{2k}r^{2k}$, where: c_{2k} is the volume of the unit ball in $\boldsymbol{R}^{2k}$, $B(z, r)$ is ball about z of radius r, and $M_\Omega(u) \equiv \sup\{|u(\varphi)| : \varphi$ is a smooth $2k$ form with compact support in Ω and $\|\varphi\|_\infty \leqq 1\}$ denotes the $2k$-*volume* or *mass* of u on Ω. For a proof that the above limit exists see Lelong [**18**]. For example, if $V = \{z \in \boldsymbol{C}^2 : z_1^2 - z_2^3 = 0\}$ and $u = [V]$ then $\Theta_2(u, z)$, the density of u at z, is equal to zero on $\boldsymbol{C}^2 - V$, one on the manifold points $V - \{0\}$, and two at $z = 0$. The integer $\Theta_{2k}([V], z)$ is, in fact, always the multiplicity of V at z (Draper [**3**]).

STRUCTURE THEOREM I. *Given a current u on an open subset of $\boldsymbol{C}^n$ which satisfies* (1), (2), *and* (3) *above then u is a holomorphic k-chain.*

See Harvey and Shiffman [**11**] for the proof. Various results of geometric measure theory are employed in this proof. One of the important steps (assume $k = n-1$) is to construct a meromorphic function f with $(i/\pi)\,\partial\bar\partial \log|f| = u$.

If $u = \sum n_j[V_j]$ is a positive holomorphic k-chain, then in addition to properties (1), (2), and (3) above, the following holds:

(4) u is positive.

By definition this means that for each smooth compactly supported function $\varphi \geqq 0$ and for each choice of linear coordinates $z = (z_1, \cdots, z_n)$, the quantity $u(\varphi(i/2)\, dz_1 \wedge d\bar z_1 \wedge \cdots \wedge (i/2)\, dz_k \wedge d\bar z_k)$ is greater than or equal to zero.

COROLLARY 1.1. *Under the hypothesis of Theorem* I *if u also satisfies* (4) *then u is a positive holomorphic k-chain.*

This very important special case of Theorem I was conjectured by Lelong [**17**] and is due to King [**14**].

One of the most interesting applications of Theorem I, which is not also a consequence of the Corollary, is a theorem of Lawson and Simons [**15**]. They prove that every stable current on complex projective n-space is a holomorphic chain. See [**11**] for other applications. (In particular, note the uniqueness result, Theorem 3.6, for a special class of Plateau problems in $\boldsymbol{C}^n$.)

A current u of the form $\sum c_j[V_j]$ with $\{V_j\}$ as above and each c_j a positive real number is called a *positive holomorphic k-chain with real coefficients*. These currents can be characterized as follows (see Harvey and King [**8**]).

THEOREM 1.2. *Suppose u is a current defined on an open subset of $\boldsymbol{C}^n$ which is positive, of bidimension k, k, d-closed, and $\Theta_{2k}(u, z)$ is bounded* *below by a positive constant on each compact subset of the support of u. Then u is a positive holomorphic k-chain with real coefficients.*

Although this theorem naturally belongs in this section, its proof (entirely unlike that of Theorem I) depends on a fundamental result of Bombieri [**1**] and [**2**] which is the basis for structure Theorem II of the next section.

2. **Density points of positive, d-closed currents of bidimension k, k.** The currents which satisfy: (1) bidimension k, k (bidegree $n-k$, $n-k$), (2) d-closed, and (4) positive, need not correspond to integration over subvarieties. For example $(i/2)\, \partial\bar\partial |z|^2 = \sum_{j=1}^{n} (i/2)\, dz_j \wedge d\bar z_j$, $i\, \partial\bar\partial \log(1+|z|^2)$, and $i\, \partial\bar\partial \log|z|^2$ $(n>1)$, are d-closed, positive currents of bidegree 1, 1 (bidimension $n-1, n-1$) which do not correspond to integration over subvarieties. More generally if φ is any plurisubharmonic function then $i\, \partial\bar\partial\varphi$ is a positive current (which is also d-closed and of bidegree 1, 1). In fact this can be taken as the definition of a plurisubharmonic function. Suppose φ is a distribution and

$$i\, \partial\bar\partial\varphi = \sum \frac{\partial^2 \varphi}{\partial z_j\, \partial \bar z_k}\, i\, dz_j \wedge d\bar z_k$$

is positive. (For 1,1 currents $u = \sum u_{jk} i\, dz_j \wedge d\bar z_k$ is positive if and only if $\sum u_{jk} \lambda_j \bar\lambda_k$ is a positive measure for each $\lambda \in \boldsymbol{C}^n$.) One can prove that φ is locally integrable and that if one defines φ pointwise by $\tilde\varphi(z) = \operatorname{ess\,lim}_{w\to z} \varphi(w)$, then $\tilde\varphi$ is classically plurisubharmonic. See Lelong [**18**] or Vladimirov [**25**] for a full discussion.

Locally every d-closed positive current of bidegree 1, 1 arises as in the above discussion. That is, given such a current u on the ball $B(0, r)$ in $\boldsymbol{C}^n$ there exists a solution φ to $i\, \partial\bar\partial\varphi = u$ (see [**11**] for example).

The present section is concerned with the points of high concentration, or density, of a positive current u. Interestingly, for positive, d-closed currents u,

$$\Theta_{2k}(u, z) = \lim_{r\to 0^+} \frac{M_{B(z,r)}(u)}{c_{2k} r^{2k}}$$

not only exists, but is the limit of a function which is decreasing as r decreases (see Lelong [**18**, Proposition 10], and Federer [**4**, Theorems 5.4.3 and 5.4.19]). Consider the example $u = (i/\pi)\, \partial\bar\partial \log|z|$ mentioned above. Then $\Theta_{2n-2}(u, z) = 0$ for $z \in \boldsymbol{C}^n - \{0\}$ and $\Theta_{2n-2}(u, 0) = 1$.

The next structure theorem is due to Siu [**22**]. This result was conjectured by Harvey and King [**8**].

STRUCTURE THEOREM II. *Suppose u is a positive, d-closed current of bidimension k, k (bidegree $n-k, n-k$) on an open subset of C^n. Then for each $c>0$, $E_c=\{z:\Theta_{2k}(u, z)\geqq c\}$ is a complex subvariety of dimension $\leqq k$.*

This structure theorem depends on a fundamental result of Bombieri [**1**] and [**2**].

STRUCTURE THEOREM II′. *Suppose φ is a plurisubharmonic function on the ball $B(0, r)$ in C^n. Then for each $c>0$ the set $I_c=\{z\in B(0, r): e^{-\varphi/c}$ is not integrable in any neighborhood of $z\}$ is a proper subvariety of $B(0, r)$.*

Bombieri [**2**] also made the following estimates.

THEOREM 2.1. *Suppose φ is given as in the above theorem and let $u=(i/\pi)\,\partial\bar\partial\varphi$ on $B(0, r)$. Then $E_{2nc}\subset I_c\subset E_{\gamma c}$ for each $c>0$, where γ is a constant depending on n.*

These results are used in [**8**] to prove Theorem 1.2 of the last section.

REMARK. Skoda [**23**, Proposition 7.1] has shown that γ can be chosen equal to 2. This result, $I_c\subset E_{2c}$, is sharp since for $\varphi=\log|z_1|^2$ and $u=(i/\pi)\,\partial\bar\partial\varphi=2[(z_2,\cdots, z_n)$ hyperplane], $e^{-\varphi}=|z_1|^{-2}$ is not integrable near the origin while u has density 2 at the origin.

Skoda [**23**] obtained the following very important generalization of Theorem 2.1 from positive currents of bidimension $n-1, n-1$ to positive currents of general bidimension.

THEOREM 2.2. *Suppose u is a positive, d-closed current of bidimension k, k (bidegree p, p where $p+k=n$) defined near the origin in C^n. Then there exists a plurisubharmonic function φ such that $E_c(u)\subset I_1(\varphi)\subset E_{pc/n}(u)$.*

In other words, $E_c(u)$ is contained in a subvariety $I_1(\varphi)$ which is not much bigger than $E_c(u)$ in the sense that $I_1(\varphi)$ is contained in $E_{pn/c}(u)$.

Siu's proof of Structure Theorem II proceeds from the above results. Let me illustrate one of the ideas of the proof by considering the following special case. Suppose u is a positive d-closed current of bidegree 1, 1 (bidimension 1, 1) defined in a neighborhood of the origin in C^2 and assume $c>0$ is given. By Theorem II′ and Theorem 2.1 above, E_c is contained in a complex curve V near the origin. For the sake of simplicity assume that V is a connected complex manifold. If $E_c=V$ the proof is complete, so assume that $E_c\subsetneq V$, or equivalently that $c'=\inf\{\Theta_2(u, z): z\in V\}$ is $<c$. First consider the case where $c'=0$. By Theorem 2.1 and the Remark we have $E_c\subset I_{c/4}\subset E_{c/2}$ Choose $z\in V$ with $\Theta_2(u, z)<c/2$. Then $z\notin E_{c/2}$ and hence $z\notin I_{c/4}$. Therefore $V\cap I_{c/4}$ is a complex subvariety of dimension zero which contains E_c. This proves that if $c'=0$ then E_c is a finite point set (i.e., a zero dimensional subvariety). Finally, assume $c'>0$ and let $v=u-c'[V]$. Suppose for the moment we have shown that v is positive. Then

$\Theta_2(u, z) \geqq c$ implies that $\Theta_2(v, z) = \Theta_2(u, z) - c' \geqq c - c'$, that is $E_c(u) \subset E_{c-c'}(v)$. By the argument given above applied to v, $E_{c-c'}(v)$ is a zero dimensional subvariety. Therefore, if $E_c \subsetneq V$ then E_c is a zero dimensional subvariety. To complete the proof we must show that $u - c'[V]$ is positive. This is a result of Siu of independent interest which holds in greater generality.

PROPOSITION 2.3. *Suppose u is a d-closed positive current of bidimension k, k, and that V is a pure k dimensional subvariety with irreducible components $\{V_i\}$. Let $c_i = \inf\{\Theta_{2k}(u, z) : z \in V_i\}$. Then $u - \sum c_i[V_i]$ is positive.*

A sketch of the proof is given for $k = n-1$ (the case needed above). For simplicity assume V is irreducible. The general case is reduced to this case. Define a measure μ by $\mu(\Omega) = M_\Omega(u)$ (μ is the volume measure of u). Let σ denote $2n-2$ dimensional volume measure induced by V (i.e., $\sigma(\Omega) = M_\Omega([V])$, which is the same as the $2n-2$ Hausdorff measure of $\Omega \cap V$). Since $\Theta_{2n-2}(u, z) \geqq c$ on V, a generalization of the Lebesgue differentiation theorem (Federer [**4**, 2.10.19(3)]) says that $\mu - c\sigma$ is a positive measure on V (and hence a positive measure). Next solve $i\,\partial\bar{\partial}\varphi = u - c[V]$. Then φ is plurisubharmonic outside V. One can show that $\frac{1}{4}\Delta\varphi = \mu - c\sigma$ (see [**18**] or [**4**] for example). Since $\mu - c\sigma$ is a positive measure φ must be subharmonic. Therefore φ is locally bounded above. Since φ is plurisubharmonic outside V and locally bounded above across V, it follows that φ is plurisubharmonic (see, for example, Harvey [**6**, part c of the theorem on p. 132]). Therefore $u - c[V] = i\,\partial\bar{\partial}\varphi$ is positive.

By using both Proposition 2.3 and Structure Theorem II a strengthened version of Theorem 1.2 can be obtained.

THEOREM 2.4. *Suppose u is a positive, d-closed, current of bidegree $(n-k, n-k)$ (bidimension k, k) on an open subset of $\mathbf{C}^n$. There exist irreducible subvarieties $\{V_j\}$ and positive constants c_j such that $u = \sum c_j[V_j] + v$ where $\sum c_j M_\Omega([V_j]) < \infty$ for each relatively compact Ω and where v is positive with the complex varieties $E_c(v)$ of dimension $\leqq k-1$. The above representation is unique.*

PROOF. Let $c_1 = \sup\{c : \dim_{\mathbf{C}} E_c = k\}$ and let V_1 denote the union of the components of E_{c_1} of dimension k. By definition $E_{c_1} = \bigcap_{c<c_1} E_c$, which implies that $E_{c_1} = E_c$ for some $c < c_1$ (see, for example, Narasimham [**19**, Theorem 2, p. 70]). This proves that E_{c_1} is of dimension k, or that $V_1 \neq \varnothing$, unless $c_1 = 0$. By Proposition 2.3, $u_1 \equiv u - c_1[V_1]$ is positive. Let $c_2 = \sup\{c : \dim_{\mathbf{C}} E_c(u_1) = k\}$ and let V_2 denote the union of the components of $E_{c_2}(u_1)$ of dimension k. As above, either $c_2 = 0$, in which case the proof is complete, or $u_2 \equiv u - c_1[V_1] - c_2[V_2]$ is positive. Continuing we have $u_N = u - \sum_{j=1}^N c_j[V_j]$ is positive. One can easily show that the coefficients of the currents u_N (which are measures) converge weakly in measure. Let v denote the limit of the u_N. Then v is positive, and d-closed. Also since

$$M_\Omega\left(\sum_1^N c_j[V_j]\right) = M_\Omega(u - u_N) = M_\Omega(u) - M_\Omega(u_N) \leqq M_\Omega(u) < \infty,$$

$M_\Omega(u_N - v) = M_\Omega(\sum_{N+1}^\infty c_j[V_j])$ converges to zero; that is, u_N converges to v is the mass norm.

Bombieri's original use of a global version of Theorem II′ in "algebraic values of meromorphic maps" [1] provides a fascinating application of the results in this section. Siu uses his results in this section to prove an extension theorem for meromorphic maps conjectured by Griffiths [5]. (See Siu [21] for the full strength of the result and the proof.)

THEOREM 2.5. *Suppose A is a subvariety of a complex manifold X of a codimension $\geqq 2$, and that Y is a compact Kähler manifold. Every meromorphic map f from $X-A$ to Y extends to a meromorphic map f from X to Y. (Cf. Griffiths* [5], *Shiffman* [20], *and Harvey* [7].)

3. **Recognizing boundaries of complex varieties.** First suppose that a compact oriented, $2k-1$ dimensional smooth submanifold M of C^n is the boundary of a complex k dimensional manifold V. Then $T_z(V) = T_z(M) \oplus [\nu]$ where $[\nu]$ is the real line spanned by the normal to M (with respect to V) at z. Therefore, the orthogonal complement in the complex vector space $T_z(V)$ of the complex line spanned by ν, is a complex vector space of complex dimension $k-1$ lying in $T_z(M)$. That is

$$\dim_C(T_z(M) \cap iT_z(M)) = k - 1 \quad \text{for all } z \in M.$$

If this condition is satisfied M will be called *maximally complex*. (Analogously with §1 one can show that the current $u=[M]$ is of bidimension $(k, k-1)$ union $(k-1, k)$ if and only if M is maximally complex.)

STRUCTURE THEOREM III. *Let M be a compact, orientable, $(2k-1)$ dimensional real submanifold of C^n of class C^r with $k, r \geqq 2$. If M is maximally complex, then M is the boundary (as a current) of a uniquely determined bounded complex subvariety V of $C^n - M$.*

For details about boundary regularity and further results see the announcement [9]. Detailed proofs are to appear in Harvey and Lawson [10].

The analogue of Theorem III for $k=1$ (i.e., M a real curve) is a known result. It is obtained by replacing the vacuous hypothesis that M be a maximally complex real curve by the condition that $\int_M \omega = 0$ for all holomorphic 1-forms on C^n. The question of which one dimensional real curves bound complex curves of one dimension has received a lot of attention beginning with fundamental results of Wermer [26] dealing with real-analytic curves. (In most of the works the focus is on finding analytic structure in the spectrum of function algebras.)

Theorem III includes, as the special case where M is the graph of a function defined on the boundary of an open set in C^{n-1}, Bochner's infinitesimal version of Hartogs' phenomenon (cf. Hörmander [12, Theorem 2.3.2′ and Theorem 2.6.13]).

See Hunt and Wells [13] for some local extension results employing the connection made in [10] between extending manifolds and extending functions.

REFERENCES

1. E. Bombieri, *Algebraic values of meromorphic maps*, Invent. Math. **10** (1970), 267–287. MR **46** #5328.

2. ———, *Addendum to my paper "Algebraic values of meromorphic maps"*, Invent. Math. **11** (1970), 163–166.

3. R. N. Draper, *Intersection theory in analytic geometry*, Math. Ann. **180** (1969), 175–204. MR **40** 403.

4. H. Federer, *Geometric measure theory*, Die Grundlehren der math. Wissenschaften, Band 153, Springer-Verlag, New York, 1969. MR **41** #1976.

5. P. A. Griffiths, *Two theorems on extensions of holomorphic mappings*, Invent. Math. **14** (1971), 27–62. MR **45** #2202.

6. R. Harvey, *Removable singularities and structure theorems for positive currents*, Proc. Sympos. Pure Math., vol. 23, Amer. Math. Soc., Providence, R.I., 1973, pp. 129–133.

7. ———, *A result on extending positive currents*, Amer. J. Math. (to appear).

8. R. Harvey and J. P. King, *On the structure of positive currents*, Invent. Math. **15** (1972), 47–52. MR **45** #5409.

9. R. Harvey and B. Lawson, *Boundaries of complex analytic varieties*, Bull. Amer. Math. Soc. **80** (1974), 180–183.

10. ———, *Boundaries of complex analytic varieties* (in preparation).

11. R. Harvey and B. Shiffman, *A Characterization of holomorphic chains*, Ann. of Math. (to appear).

12. L. Hörmander, *An introduction to complex analysis in several variables*, Van Nostrand, Princeton, N.J., 1966, MR **34** #2933.

13. L. R. Hunt and R. O. Wells, Jr., *Holomorphic extension for non-generic* C.R. *submanifolds*, Proc. Sympos. Pure Math., vol. 27, Amer. Math. Soc., Providence, R.I. (to appear).

14. J. King, *The currents defined by analytic varieties*, Acta Math. **127** (1971), 185–220.

15. H. B. Lawson and J. Simons, *On stable currents and their application to global problems in real and complex geometry*, Ann. of Math. **98** (1973), 427–450.

16. P. Lelong, *Intégration sur un ensemble analytique complexe*, Bull. Soc. Math. France **85** (1957), 239–262. MR **20** #2465.

17. ———, *Propriétés métriques des ensembles analytiques complexes*, Séminaire Lelong, 6e année: 1965/66, no. 2, Secrétariat mathématique, Paris, 1966. MR **36** #4941.

18. ———, *Fonctions plurisousharmoniques et formes différentielles positives*, Gordon and Breach, New York (distributed by Dunod, Paris), 1968. MR **39** #4436.

19. R. Narasimhan, *Introduction to the theory of analytic spaces*, Lecture Notes in Math., no. 25, Springer-Verlag, New York and Berlin, 1966. MR **36** #428.

20. B. Shiffman, *Extension of positive line bundles and meromorphic maps*, Invent. Math. **15** (1972), 332–347.

21. Y.-T. Siu, *Analyticity of sets associated to Lelong numbers and the extension of meromorphic maps*, Bull. Amer. Math. Soc. **79** (1973), 1200–1205.

22. ———, *Analyticity of sets associated to Lelong numbers and the extension of closed positive currents* (to appear).

23. H. Skoda, *Sous-ensembles analytiques d'ordre fini ou infini dans* C^n, Bull. Soc. Math. France **100** (1972), 353–408.

24. G. Stolzenberg, *Volumes, limits, and extensions of analytic varieties*, Lecture Notes in Math., no. 19, Springer-Verlag, Berlin and New York, 1966. MR **34** #6156.

25. V. S. Vladimirov, *Methods of the theory of functions of several complex variables*, "Nauka", Moscow, 1964; English transl., M.I.T. Press, Cambridge, Mass., 1966. MR **30** #2163; **34** #1551.

26. J. Wermer, *The hull of a curve in* C^n, Ann. of Math. (2) **68** (1958), 550–561. MR **20** #6536.

27. H. B. Lawson, *Minimal varieties in real and complex geometry*, Montreal Lecture Notes.

Current address: Department of Mathematics, Rice University, Houston, Texas 77001