In [2] and [3] a condition on partially ordered linear algebras (pola’s) is defined, and it is shown that Dedekind \(\sigma\)-complete polas satisfying this condition have many of the properties of function spaces. Using a theorem of H. Nakano we can show, even without the hypothesis that the pola is Dedekind \(\sigma\)-complete, that any such pola is isomorphic to a pola of continuous, almost-finite, extended-real-valued functions. If \(A\) is a pola with multiplicative identity 1 the condition mentioned is:

\[ P_1. \text{ If } x \in A \text{ and } x \geq 1, \text{ then } x \text{ has an inverse and } x^{-1} \geq 0. \]

**Theorem.** In order for an Archimedean pola \(A\) with identity 1 to be isomorphic to a pola of continuous, almost-finite, extended-real-valued functions on a compact Hausdorff space \(X\), it is sufficient that \(P_1\) hold for \(A\). The condition is necessary also if \(A_1 = \{y \in A: \text{ there exists } a \in R^+ \text{ with } -a1 \leq y \leq a1\}\) is complete in the order unit norm derived from 1 and if the image of \(A_1\) separates points in \(X\).

**Proof.** The standard completion procedure for Archimedean ordered linear spaces shows that \(A\) is isomorphic with an order dense subspace \(\hat{A}\) of a Dedekind complete linear lattice \(D\). In [4, p. 150] it is shown that the multiplication on \(\hat{A}\) can be extended to \(D\) in such a way that \(D\) is a pola if the following continuity condition is satisfied: For every subset \(B\) of \(A\), \(\inf B = 0\) implies \(\inf(aB) = \inf(Ba) = 0\) for all positive elements \(a\) in \(A\). Given \(P_1\), multiplication by \((a+1)^{-1}\) shows this condition is satisfied. Thus \(D\) is a linear lattice pola and the order density of \(\hat{A}\) shows (since 1 is easily seen to be a weak order unit for \(A\)) that the image of 1 is a weak order unit for \(D\). Now \(D\) (and hence \(A\)) has a representation of the type desired by [1, Corollary, p. 625].

To prove the second statement we note that the assumptions, together with the Stone–Weierstrass theorem, give the result that if \(A \rightarrow \hat{A}\) is the isomorphism then \(\hat{A}_1 = C(X)\). Then, given any \(x\) in \(A\) such that \(x \geq 1\),

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we can define an \( f \) in \( C(X) \) by \( f(t) = 1/\|x(t)\| \) for all \( t \) in \( X \) (with \( 1/\infty \) set equal to 0). Then there exists \( z \) in \( A_1 \) such that \( z = f \) and it is clear that \( z = x^{-1} \) and \( z \geq 0 \).

Note that it is not enough to know that \( A \) separates points of \( X \) to conclude that \( A_1 \) does. This shows the need for the separation assumption. Also, it is easy to see that if \( A \) is Dedekind \( \sigma \)-complete, then \( A_1 \) is complete in the order unit norm, so this case is included.

An immediate consequence of this theorem is the useful result that if a pola is Archimedean, has an identity, and satisfies \( P_1 \), then it is necessarily commutative.

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