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JOSEPH L. WALSH IN MEMORIAM

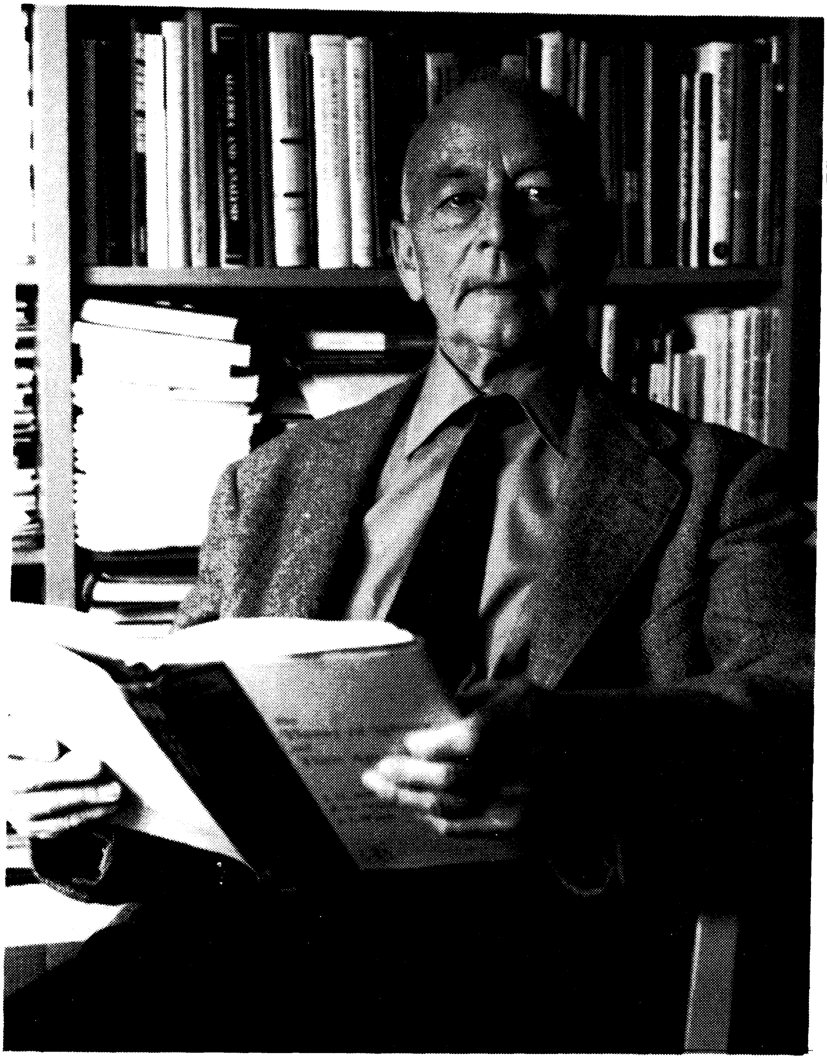
BY MORRIS MARDEN

Threescore years and ten is the biblical measure of a man's normal life span. Yet in modern times his surpassing of this bound is not unusual. However, it becomes noteworthy when he is blessed with undiminished physical and mental vigor. This was true of Joe Walsh. "His spirit was marvelous until the end, and he spoke with gratitude for his many productive years."[1]

Joseph Leonard Walsh died on December 10, 1973 at the age of seventy eight years in his home at University Park, Maryland. This site is not far from where he was born on September 21, 1895, as son of Reverend and Mrs. John Leonard Walsh. Most of his academic life as student, teacher and scholar was spent at Harvard University. In 1916 Harvard awarded him the S.B. degree, summa cum laude, and at the same time a Sheldon Traveling Fellowship for study at the Universities of Chicago and Wisconsin. On his return to Harvard in 1917, Walsh began some studies under Maxime Bôcher, but their progress was interrupted by World War I and his enlistment as an ensign in the U.S. Navy. In 1920 Harvard granted him a Ph.D. and also a second Sheldon Traveling Fellowship, this time for study in Paris under Paul Montel. Back from Europe in 1921 he joined the Harvard faculty, but in 1925 took a leave-of-absence for a year's research at Munich under Carathéodory. Returning again to Harvard, he was promoted through the ranks to a full professorship in 1935 and served as department chairman from 1937 to 1942. In the latter year he was recalled to active duty in the U.S. Navy as a lieutenant commander. When he returned to Harvard in 1946, he was appointed to the prestigous Perkins Professorship, which he held until his retirement in 1966. A semester earlier he began a research professorship at the University of Maryland, in which position he remained fully active, working with Ph.D. and post doctoral students, until a few months before his death.

During Walsh's lifetime he received many academic and nonacademic honors. Among them was his election in 1936 to the National Academy of Science, and in 1937 to the vice-presidency of the American Mathematical Society. He was elected for a two-year term as president of the Society in 1949. This was a crucial period for the Society when it was experiencing

[1] Letter from Mrs. Joseph L. Walsh.



JOSEPH L. WALSH (1895–1973)

growing pains due to a rapid increase in membership and in research publications. It was the period in which the Society created the post of Executive Director and moved its headquarters to Providence. Also during this period Walsh served as chairman of the organizing committee for the International Mathematical Congress, held in Cambridge during August 1950, the first since World War II. At about this time Walsh was recipient of a nonmathematical honor—promotion to the rank of captain in the U.S. Naval Reserve. In his later years Walsh was twice honored by the dedication of volumes of mathematical journals: SIAM J. (2) **3** (1966) on his seventieth birthday in 1965, and J. Approximation Theory **5** (1972) on his seventy-fifth birthday in 1970.

These mathematical honors were well deserved in view of the quantity and quality of his original research. Starting with his first publication in 1916, while still an undergraduate, he wrote, singly or jointly with students and others, a total of 279 research, expository, and book review articles as well as seven books. Though these papers covered a wide range of topics, they were, broadly speaking, concerned with four general areas:

(I) The relative location of the zeros of pairs of rational functions such as a polynomial and its derivative.

(II) Zeros and topology of extremal polynomials.

(III) The critical points and level lines of Green's function and other harmonic functions.

(IV) Interpolation and approximation of continuous, analytic, or harmonic functions.

Regarding the general area (I), this was Walsh's first main research interest. His doctoral thesis was entitled *On the roots of the jacobian of two binary forms*. It was written under the guidance of Maxime Bôcher who had proved that if F and G are binary forms of the same degree and if all the zeros a_j of F lie in a circular region A and all the zeros b_j of G lie in a circular region B with $B \cap A = \varnothing$, then all the zeros c_k of the jacobian $J(F, G)$ lie in $A \cup B$. Like Bôcher, Walsh used geometric and physical methods, interpreting the c_k as equilibrium points in the field due to positive masses at the points a_j and negative masses at the points b_j with an inverse distance force law. Walsh's results are generalizations of the Lucas theorem that the convex hull of the zeros of a polynomial f contains all the critical points of f. These results are described in Walsh's papers and in M. Marden, *Geometry of polynomials*, 2nd ed., Math Surveys, no. 3, Amer. Math. Soc., Providence, R.I., 1966, MR **37** #1562. The most striking of these results are the following three:

(1) If an nth degree polynomial f has n_1 zeros in a disk $|z-c_1| \leqq r_1$ and the remaining $n_2 = n - n_1$ zeros in the disk $|z-c_2| \leqq r_2$, then any critical

point of f not in one of these disks lies in a third, "average" disk

$$|z-(n_2c_1+n_1c_2)/n| \leqq (n_2r_1+n_1r_2)/n.$$

(2) If C_1, C_2 and C_3 are disjoint circular regions and if a rational function f has in the extended plane all its zeros in $C_1 \cup C_2$ and all its poles in C_3, then any critical point of f not in $C_1 \cup C_2 \cup C_3$ lies in a circular region C_4. The boundary ∂C_4 of C_4 is the locus of the point z_4 defined by the cross ratio $(z_1, z_2, z_3, z_4)=\text{const}$ when z_1, z_2, z_3 vary independently on the circles ∂C_1, ∂C_2, ∂C_3 respectively.

(3) Let the form $\Phi(z_1, z_2, \cdots, z_n)$ be of degree one in each z_j, and of total degree n and symmetric in the set $z_1, z_2, \cdots, z_n$. Let C be a circular region containing the n points $z_j=z_{j0}$, $j=1, 2, \cdots, n$. Then in C there exists at least one point ζ such that $\Phi(\zeta, \zeta, \cdots, \zeta)=\Phi(z_{10}, z_{20}, \cdots, z_{n0})$.

Regarding the general area (II), the methods and results were suggested in part by those in area (I). Given a closed bounded set E containing at least $n+1$ points and the class P_n of all polynomials $z^n+a_1z^{n-1}+\cdots+a_n$, an infrapolynomial p on E means a polynomial $p \in P_n$ with the property

$$\max_{z \in E} |p(z)| = \min_{q \in P_n} \max_{z \in E} |q(z)|.$$

The zeros of p play a role vis-à-vis set E similar to that of the critical points of a polynomial f vis-à-vis the zeros of f. For instance, Fejér proved that the zeros of p lie in the convex hull of E, and Fekete showed that p satisfies a form involving the points of E that is similar to the form for the logarithmic derivative of f in terms of the zeros of p. Walsh explored the subject of infrapolynomials intensively in papers which he published singly or jointly with Fekete, Motzkin, Shisha and Zedek.

Likewise the general area (III) was partly an offshoot of area (I). For example, Walsh proved that if G is the Green's function (with pole at infinity) for an unbounded region R with bounded boundary B, then all the critical points of G in R lie in the convex hull of B. Thus, the critical points of G play a role similar to the critical points of a polynomial whose zeros lie on B. In this connection Walsh also examined in detail the curvature and other characteristics of the level lines of Green's function. These theorems together with their generalizations to harmonic measures and other harmonic functions are developed in Walsh's papers and described in his book, *Critical points of analytical and harmonic functions.*

As for general area (IV), the subjects of interpolation and approximation encompass about half of Walsh's published articles as well as his now classical treatise entitled *Interpolation and approximation.* Among his many original results in this area, probably the most important are the following two:

(1) Every function continuous on a bounded Jordan arc J can be approximated on J uniformly by a polynomial in z.

(2) Every function f analytic in a Jordan region B and continuous on its closure $\bar{B}$ can be uniformly approximated on $\bar{B}$ by a polynomial in z.

The first is a generalization of Weierstrass' theorem, which requires arc J to be a closed interval of the real axis. The second is a generalization of Runge's theorem, which requires f to be analytic in $\bar{B}$. To prove this second theorem, Walsh approximated to B by a sequence of Jordan regions B_n with $\bar{B}\subset B_{n+1}\subset B_n$ for all n, and then applied Runge's theorem in $\bar{B}$ to the function $F_n(z)=f(\chi_n(z))$, where $w=\chi_n(z)$ maps B_n one-to-one conformally onto B. Walsh later extended this second theorem to sets B which are the union of a finite number of disjoint Jordan regions. Thus he paved the way for the more comprehensive theorem proved later by Mergelyan: if f is a function continuous on any closed bounded set S and analytic at all interior points of S, then it can be approximated on S uniformly by a polynomial in z.

Walsh maintained an active interest in interpolation, approximation and related topics over a period of about fifty years. Most recently he helped develop some of the fundamental theorems concerning spline interpolation and approximation both on the real line and in the complex plane. His contributions may be found in his published papers as well as in the monograph *The theory of splines and their applications*, which he wrote jointly with J. H. Ahlberg and E. N. Nilson.

In the related area of orthogonal expansions, special mention should be made of the so-called Walsh functions $\varphi_n^{(k)}(x)$. These are defined on the interval $0\leqq x\leqq 1$ by the relations $\varphi_0(x)\equiv 1$;

$$\begin{aligned}
&\varphi_1^{(1)}(x) = 1, \qquad 0 \leqq x < \tfrac{1}{2}; \qquad \varphi_1^{(1)}(x) = -1, \qquad \tfrac{1}{2} < x \leqq 1;\\
&\varphi_{n+1}^{(2k-1)}(x) = \varphi_{n+1}^{(2k)}(x) = \varphi_n^{(k)}(2x), \qquad 0 \leqq x < \tfrac{1}{2};\\
&\varphi_{n+1}^{(2k-1)}(x) = -\varphi_{n+1}^{(2k)}(x) = (-1)^{k+1}\varphi_n^{(k)}(2x-1), \qquad \tfrac{1}{2} < x \leqq 1;\\
&k = 1, 2, 3, \cdots, 2^{n-1}; \qquad n = 1, 2, 3, \cdots .
\end{aligned}$$

These functions, having some similarity to the Haar functions, were invented by Walsh in 1923 (see his paper 1923b). Now among the most widely used complete orthonormal systems, these functions serve as a valuable mathematical tool in communications engineering and other applied sciences.[2] They are treated in Harmuth's book on *Transmission of information by orthogonal functions*, 2nd ed., Springer-Verlag, New York, 1972; in a survey article by Balashov and Rubinshtein, *Series with*

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[2] Letter from T. J. Rivlin.

respect to Walsh system and their generalization, J. Soviet Math. **1** (1973), 727–763 which lists 140 references; in an article by N. J. Fine, *Encyclopaedic Dictionary of Physics*, suppl. vol. 4, Pergamon, New York, 1971; and in *Proc. Sympos. on Applications of Walsh Functions*, 1970, 1971, 1972 and 1973. A fifth such symposium was held March 18–20, 1974 at the Catholic University of America in Washington, D.C.

Not only did Walsh himself contribute to the above described areas of research, but also he introduced many of his students to these areas. This is evident from the titles of the doctoral theses written under his direction.

Walsh had altogether thirty-one Ph.D. students. The present writer succeeded in contacting nearly all of them to ask for their recollections of him as a research advisor, teacher and friend. In what follows are recorded some of this memorabilia.

Nearly all his former students agreed that, as a thesis advisor, Walsh was patient, generous and considerate, and freely gave help and encouragement. This was true, not only during the developmental stages of each thesis, but even during the years afterwards. It must have given him a great deal of satisfaction that over the years so many of his students continued to contribute to mathematical literature, and that they wrote special papers in his honor on the occasion of his seventieth and seventy-fifth birthdays.

Their remembrance of Walsh as a teacher almost always includes certain rituals accompanying each of his lectures. The opening ritual was to fling the classroom windows wide open regardless of outside temperatures and to deliver his lecture pacing back and forth on the platform while the students might be freezing in their seats. The closing ritual seemed to have been to toss his chalk into the waste basket from whatever position he ended the lecture. Besides, during the run of a lecture he was occasionally known to have used an inattentive student as a target for the chalk. Elementary calculus and introductory complex function theory were his favorite courses, but whatever the subject he prepared his lectures with meticulous care and presented them in a deep, musical voice.

Nearly all his former students were impressed by his love of walking. Regardless of the weather he would hike the mile and a half (or so) from his home near Fresh Pond to Widener Library or Seaver Hall. When he accepted the appointment to the University of Maryland, his first prerequisite for a new home was that it be within walking distance of the University.

Some of his former students recollect his sense of humor and his enjoyment of practical jokes. For example, at the 1948 summer meetings of the Society in New Haven, a group photograph was being taken with the mathematicians seated on wide circular bleachers. As the camera rotated, Walsh got his companion to run with him from one end of the bleachers to the other and thus they appeared twice in the same composite photograph.[3] As another instance, a former student who felt himself a novice in teaching sought pedagogic advice from Walsh, to which the latter gave the terse reply: "Always start writing in the upper left-hand corner of the blackboard".[4,5]

In summary, Walsh may be characterized as having had a strong sense of duty to religion, country and chosen work, as well as a love of art and music and a Thoreau-like love of nature.[6] Above all, he was a resolute, hard worker, known to have spent long hours daily in his Widener study. He explored his problems thoroughly along all possible paths and byways, usually reporting his discoveries in a succession of papers. Often years later he revisited the same problems, searching for nuggets that he may have missed on his earlier explorations. When asked on one occasion how he managed to keep up the terrific pace of publication, Walsh pointed to the self-portrait of an artist showing Death standing nearby and the artist working to complete as much as possible in the little time that remained.[3] The verdict of history will surely be that Walsh did win this race against time. He did leave a permanent imprint upon the mathematics of this century and especially upon the men and women whose mathematical careers he helped to launch.

In the words of the poet, Henry Wadsworth Longfellow, [*Charles Sumner*, stanza 9]

"So when a great man dies
For years beyond our ken
The light he leaves behind him lies
Upon the paths of men."

Papers by Joseph L. Walsh

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[3] Letter from E. N. Nilson.

[4] Letter from T. J. Rivlin.

[5] Other anecdotes may be found in the two articles: D. V. Widder, *Joseph Leonard Walsh*, J. SIAM Numer. Anal. **3** (1966), 171–172; M. Marden, *Homage to Walsh*, J. Approximation Theory **5** (1972), ix–xiii.

[6] Letter from Maurice Heins.

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Walsh's Ph.D. Students

(With journal reference to some or all their thesis results)

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