We announce some new results about multipliers and ideal theory of $A$-Segal algebras and their relative completions. Complete details are to appear elsewhere [3], [4]. The results about multipliers (Theorem 6) represent work done jointly with Richard R. Goldberg [4].

**Definitions.** If $A$ is a Banach algebra, we say the subalgebra $B \subseteq A$ is an $A$-Segal algebra provided $B$ is a dense left ideal of $A$, $B$ is a Banach algebra with respect to a norm $\| \|$$_B$, the injection of $B$ into $A$ is continuous, and multiplication is (jointly) continuous on $A \times B$ into $B$. We shall always suppose that $A$ does not have an identity.

The relative completion of $B$ with respect to $A$, denoted $\widehat{B}^A$, is defined by

$$\widehat{B}^A = \bigcup_{n>0} S_B(\eta)^A,$$

where $S_B(\eta) = \{ f \in B| \|f\|_B < \eta \}$ and $\overline{E}^A$ is the $A$ closure of $E$. For $f \in \widehat{B}^A$ we define $\| f \|$ by

$$\| f \| = \inf \{ \delta | f \in S_B(\delta)^A \}.$$

**Theorem 1.** If $B$ is an $A$-Segal algebra, then $\widehat{B}^A$ (with norm $\| \|$) is an $A$-Segal algebra. Furthermore, if $B$ has right approximate units which are bounded in the $A$-norm, then $B$ is a closed left ideal of $\widehat{B}^A$ and the embedding of $B$ into $\widehat{B}^A$ is isometric if the approximate units have $A$-norm one.

In case $A$ and $B$ share common right approximate units of $A$-norm one, then $\widehat{B}^A$ has a rather simple description which permits a straightforward proof of all of the assertions of Theorem 1. Such is the case when $A = L^1(G)$ and $B$ is an ordinary Segal algebra [6, p. 16]. Indeed,

**Theorem 2.** With $A$ and $B$ as in the preceding paragraph and $U$ denoting (a set of) common right approximate units, we have

\[ f \in \widetilde{B}^A \Leftrightarrow M \equiv \text{Sup} \{ \|u \ast f\|_B \mid u \in U\} < \infty, \]
and in this case \( \|f\| = M. \)

From here on we suppose that \( A \) and \( B \) have common right approximate units of \( A \)-norm one. The following theorem, which has the assertion of the second sentence in Theorem 1 as a consequence, is of independent interest.

**Theorem 3.** \( S_B(\delta) = \overline{S_B(\delta)^A} \cap B; \) in particular, if \( B = \widetilde{B}^A \), then \( S_B(\delta) = \overline{S_B(\delta)^A}. \)

**Definition 4.** We say \( B \) is singular provided \( B \neq \widetilde{B}^A. \)

Perhaps the simplest example of a singular \( A \)-Segal algebra and its relative completion is the pair \((C(G), L^\infty(G))\), where \( G \) is an infinite compact group and \( A = L^1. \) Additional examples of singular Segal algebras are given in \([3]\) and \([4]\); a more detailed analysis of singularity is given in \([3]\).

Some results which are useful for an analysis of multipliers and the ideal theory of \( A \)-Segal algebras and their relative completions are summarized in

**Theorem 5.** (1) If \( B \) is a closed ideal in the \( A \)-Segal algebra \( B \), then \( B_1 \subseteq \widetilde{B}^A. \) Let \( U \) denote right approximate units for \( B. \) (2) If \( f \in \widetilde{B}^A \), then \( f \in B \Leftrightarrow \) given any \( \epsilon > 0 \) there exists \( u(\epsilon, f) \equiv u \in U \) so that \( \|uf - f\| < \epsilon. \) (3) \( \widetilde{A}^B \subseteq B \) and, hence, \( \widetilde{B}^A : \widetilde{B}^A \subseteq B. \) We thus see that \( \widetilde{B}^A \) fails to factor if \( B \) is singular.

We now specialize to the case where \( A = L^1(G) \), and \( B = S(G) \) is a symmetric Segal algebra as defined by H. Reiter \([6, \text{p. 17}]\). Here, \( G \) denotes a locally compact nondiscrete group. The (multiplier) algebra of bounded linear operators from \( L^1(G) \) into \( S(G) \) (\( \widehat{S}L^1(G) \)) for which \( T(f \ast g) = f \ast Tg \) is denoted \((L^1, S) ((L^1, \widehat{S}L^1)). \)

**Theorem 6.** Let \( \{e_\alpha\} \) be a bounded approximate identity for \( L^1(G). \) For a measure \( \mu \in M(G) \) the following three conditions are equivalent:
(1) \( \text{Sup}_\alpha \|e_\alpha \ast \mu\|_S < \infty; \) (2) \( \mu \in (L^1, S); \) (3) \( \mu \in (L^1, \widehat{S}L^1). \)

Furthermore, if \( (L^1, S) \subseteq L^1(G) \), then \( (L^1, S) \) is isometrically isomorphic with \( \widehat{S}L^1. \)

For our final theorems we require that \( G \) be an infinite compact group. All unexplained notation may be identified from the analogous results in \([5]\).

**Theorem 7** \([5, \text{38.9, p. 453}]\). Let \( S(G) \) be a singular Segal algebra. Let
Let $F$ be any subset of $\Sigma$. Let $F$ be a closed linear subspace of $\tilde{S}^{L^1}(G)$ for which $F \cap S(G) = S_p(G)$ and $F \subseteq \tilde{S}^{L^1}(G)$. Then $F$ is a closed two-sided ideal in $\tilde{S}^{L^1}(G)$; conversely, all closed two-sided ideals in $\tilde{S}^{L^1}(G)$ have this form. Furthermore, the quotient algebra $\tilde{S}^{L^1}(G)/S(G)$ is a zero algebra. The closed two-sided ideals in $\tilde{S}^{L^1}(G)$ for which the quotient algebra is a zero algebra are exactly the closed linear subspaces of $\tilde{S}^{L^1}(G)$ that contain $S(G)$.

**Theorem 8.** Suppose $S(G)$ is a singular Segal algebra. For each $\sigma \in \Sigma$, $\tilde{S}^{L^1}_{\{\sigma\}}(G)$ is a regular maximal proper two-sided ideal in $\tilde{S}^{L^1}(G)$. If $M$ is a nonzero bounded linear functional on $\tilde{S}^{L^1}(G)$ which vanishes on $S(G)$, then $M^{-1}(0)$ is a closed maximal proper two-sided ideal in $\tilde{S}^{L^1}(G)$ for which $\tilde{S}^{L^1}_{\{\sigma\}}(G)/M^{-1}(0)$ is a 1-dimensional zero algebra. Every maximal closed proper two-sided ideal of $\tilde{S}^{L^1}(G)$ not of the form $\tilde{S}^{L^1}_{\{\sigma\}}(G)$ is obtained in this way.

For the ideal theory of $A$-Segal algebras with approximate identities, we refer to [1] and [2].

**References**


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