A STRONG NONIMMERSION THEOREM FOR $\mathbb{R}P^{8l+7}$

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In this paper we shall sketch the proof of a nonimmersion theorem for real projective spaces of dimension $8l + 7$ which is conjectured to be best possible. Details will appear elsewhere.

**THEOREM.** Let $\alpha(n)$ denote the number of 1's in the binary expansion of $n$. Let

$$
\beta(n) = \begin{cases} 
2\alpha(n) & \text{if } \alpha(n) \equiv 1, 2(4), \\
2\alpha(n) + 1 & \text{if } \alpha(n) \equiv 0(4), \\
2\alpha(n) + 2 & \text{if } \alpha(n) \equiv 3(4).
\end{cases}
$$

If $n \equiv 7(8)$, then $\mathbb{R}P^n \not\subset \mathbb{R}^{2n-\beta(n)}$.

This result was announced in [4] but difficulties [2] were found in the argument sketched there. It was conjectured in [4] that if $n \equiv 7(8)$, then $\mathbb{R}P^n \subset \mathbb{R}^{2n-\beta(n)+1}$. Using techniques of [1] we have proved these immersions when $\alpha(n) = 5, 6, 8, \text{ or } 9$ (unpublished), thus establishing the precise immersion dimension in these cases.

It is well known that the theorem is equivalent to showing that the map $\mathbb{R}P^n \to B\mathbb{S}p$ which classifies $(2^L - n - 1)\xi_n$ does not lift to $\tilde{B}\mathbb{S}p_{n-\beta(n)}$ [1] (where $L$ is any sufficiently large integer). We prove the nonexistence of this lifting by showing that a $bo$-secondary obstruction is nonzero with zero indeterminacy.

As in [5] we define $bo_i^4$ to be connective $\Omega$-spectrum whose $(8k + 4 - i)$th-space is $BO(8k + 4, \infty)$ localized at 2. Let $\theta: bo \to bo_i^4$ be the map inducing the Adams operation $\psi^3 - 1$, and let $bJ$ denote the...
fibre of $\theta$. Let $B^0_N = \widetilde{BSp}_N \wedge_{BSp} bo$ denote the space which was called $E^0_N$ in [1], and similarly define $B^I_J = \widetilde{BSp}_N \wedge_{BSp} bJ$.

In stable dimensions ($\leq 2N$), using techniques of [1], [5] and [6], we can form the first two stages of a $bo$-resolution of $V_N \to \widetilde{BSp}_N \to BSp$

$$
\begin{array}{c}
\vdots \\
V_N \wedge bo \\
\downarrow i \\
B^I_J \\
\downarrow c_1 \\
V_N \wedge bo^4 \\
\downarrow f \\
\Sigma V_N \wedge bo,
\end{array}
$$

where $c_1 \circ i = 1 \wedge \theta$.

Let $N = n - \beta(n)$. The theorem is proved by showing that there is a lifting of $f$ to $B^0_N$ which does not lift to $B^I_J$, and that the indeterminacy $(1 \wedge \theta)_* : [RP^n, V_N \wedge bo] \to [RP^n, V_N \wedge bo^4]$ is zero. To prove the non-lifting to $B^I_J$ we construct an $(n - 1)$-modified Postnikov tower [3], $B^I_J \to E_r \to \cdots \to E_1 \to BSp$ and show using [1, Theorem 1.8] that $RP^{n-1}$ lifts to $E_{\alpha(n)}-3$. Using the Serre spectral sequence we show that the map of 7-connected coverings $B^I_J(8, \infty) \to E_{\alpha(n)}-3(8, \infty)$ is induced through dimension $n - 1$ by a map $E_{\alpha(n)}-3(8, \infty)^{(n-1)} \xrightarrow{\widetilde{c}} Y$, and if $\widetilde{f} : RP^{n-1} \to E_{\alpha(n)}-3(8, \infty)$ is a lifting, then we show that $\widetilde{c} \widetilde{f}$ is nontrivial, so $\widetilde{f}$ does not lift to $B^I_J(8, \infty)$. This is then used to show that a lifting to $B^0_N$ does not lift to $B^I_J$.

REFERENCES


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