
BY S. Y. HUSSEINI

Communicated by Glen Bredon, October 24, 1974

In [1] Bredon showed that the complex \( X_\alpha = S^k \cup_{\alpha} D^{2m} \) has the fixed-point property with \( [\alpha] \in \pi_{2m-1}(S^k) \) being nontrivial, provided that the following condition holds.

**CONDITION (\(*\).** \( k \) is odd, and \( r = 2m - k - 1 < k - 1 \).

But \( X_\alpha \times X_\alpha \) admits a fixed-point free map if \( p \), the order of \([\alpha]\), is relatively prime to \( p' \), the order of \([\alpha']\). To show that the analogous situation holds for manifolds, let \( M_{2m} \) be a 2\( n \)-dimensional compact smooth manifold, with \( 2m < n \) and \( \pi_1(\partial M_{2m}) = \{1\} \), of the same homotopy type as \( X_\alpha \), and put \( M = M_{2m} \cup_h M_{2m} \) where \( h: \partial M_{2m} \to \partial M_{2m} \) is a diffeomorphism.

**THEOREM 1.** Suppose in addition to Condition (\(*\)) that \( r \) is not of the form \( 2^s - 2 \), and that \( p \), the order of \([\alpha]\) in \( \pi_{2m-1}(S^k) \) is greater than 2 if \( r = 0 \mod 8 \). Then the connected sum \( M \# CP^n \), of \( M \) and the complex \( n \)-projective space \( CP^n \), has the fixed-point property if \( n + 1 \) is relatively prime to both \( p \) and \( \varphi(p) \) where \( \varphi(p) \) is the Euler function of \( p \).

To prove the theorem one shows that the Lefschetz number \( L(f) \) of any map \( f: M \# CP^n \to M \# CP^n \) is given by the equation

\[
L(f) = - (\kappa + \kappa') + (\mu + \mu') + (1 + \lambda + \cdots + \lambda^n)
\]

where \( \kappa, \kappa', \mu, \mu' \) and \( \lambda \) are integers such that

\[
\kappa \kappa' = \lambda^n = \mu \mu', \quad \kappa = \mu \mod q \text{ and } \kappa' = \mu' \mod q
\]

with \( q \) being a proper divisor of \( p \). In fact \( q \) is the order of the class of \([\alpha]\) in \( \Pi_r(S)/\text{image } J \), where \( \Pi_r(S) \) is the stable \( r \)-stem \( \pi_{r+*}(S^*) \) and \( J \) the stable \( J \)-homomorphism \( \pi_r(SO) \to \Pi_r(S) \), and the conditions on \( r \) are required to ensure that \( q > 1 \) and that the congruence \( \kappa' = \mu' \mod q \) holds.

---


1While working on this paper the author was partially supported by the National Science Foundation, Grant No. GP-29538-A4.

Copyright © 1975, American Mathematical Society
THEOREM 2. With the assumptions of Theorem 1, suppose that $M$ and $M'$ are the doubles, respectively, of $M_{2m}$ and $M'_{2m}$. Then $(M \# CP^n) \times (M' \# CP^n)$ does not have the fixed-point property if $(p, p') = 1$.

To prove Theorem 2 one first retracts $(M \# CP^n) \times (M' \# CP^n)$ onto $M_{2m} \times M'_{2m}$, and then one proceeds to retract $M_{2m} \times M'_{2m}$, according to [1], onto $S^k$ considered a submanifold of the diagonal of $(M \# CP^n) \times (M' \# CP^n)$.

It is a pleasure to express my thanks to Ed Fadell for many useful conversations on this topic, and for his critical reading of an earlier manuscript which helped greatly in the final development of this work.

REFERENCE


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN 53706

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use