This is a summary of results obtained while attempting to classify the finite complexes which can be fixed-point sets of cellular actions of a given group on finite contractible CW complexes. Here, by a cellular action is meant one where the action of any group element takes the interior of any cell to the interior of some other cell, and takes a cell to itself only via the identity map. For groups of prime power order, the question has already been answered by P. A. Smith [4] and Lowell Jones [2]: if \( |G| = p^n \), a finite complex can be a fixed-point set if and only if it is \( \mathbb{Z}_p \)-acyclic.

The main tools for answering the question for groups not of prime power order are certain functions defined below, which serve as bookkeeping devices for controlling the fixed-point structure of a space with group action. Let \( G^1 \) denote the class of finite groups \( G \) with a normal subgroup \( P \triangleleft G \) of prime power order such that \( G/P \) is cyclic. A resolving function for a finite group \( G \) is defined to be a function \( \varphi : S(G) \to \mathbb{Z} \) (where \( S(G) \) is the set of subgroups) such that:

1. \( \varphi \) is constant on conjugacy classes of subgroups.
2. \( [N(H): H] \mid \varphi(H) \) for all \( H \subseteq G \).
3. \( \sum_{K \supseteq H} \varphi(K) = 0 \) for all \( H \in G^1 \).

Now define a \( G\)-resolution of a finite complex \( F \) to be any \( n \)-dimensional \((n-1)\)-connected complex \( X \) \((n \geq 2)\) such that \( G \) acts on \( X \) with fixed-point set \( F \), and such that \( H_n(X) \) is a projective \( \mathbb{Z}[G] \)-module. To any \( G \)-resolution \( X \) there corresponds a unique resolving function \( \varphi \) satisfying

\[
\chi(X^H) = 1 + \sum_{K \supseteq H} \varphi(K) \quad \text{for all } 0 \neq H \subseteq G,
\]

\[
\sum_{H \subseteq G} \varphi(H) = 0.
\]
Conversely, it has been shown that for any $G$ not of prime power order, given an integral resolving function $\varphi$ and a finite complex $F$ with $\chi(F) = \varphi(G) + 1$, there is a $G$-resolution $X$ of $F$ which realizes $\varphi$. Letting $m(G)$ denote a generator of $\{\varphi(G): \varphi$ is a resolving function for $G\}$, this proves

**Theorem 1.** If $G$ is not of prime power order, then a finite complex $F$ has a $G$-resolution if and only if $\chi(F) \equiv 1 \pmod{m(G)}$.

For any $n$-dimensional $G$-resolution $X$ of $F$, an obstruction is defined lying in the projective class group $\widetilde{K}(\mathbb{Z}[G])$: $\gamma_G(F, X) = (-1)^n [H_n(X)]$. A subgroup

$$\mathcal{B}(G) = \{\gamma_G(pt, X): X \text{ is a } G\text{-resolution of a point}\}$$

is defined. If $X_1$ and $X_2$ are two $G$-resolutions of $F$, then $\gamma_G(F, X_1) - \gamma_G(F, X_2) \in \mathcal{B}(G)$, and so there is a well-defined obstruction $\gamma_G(F) \in \widetilde{K}(\mathbb{Z}[G])/\mathcal{B}(G)$, which is zero if and only if $F$ is the fixed-point set of an action of $G$ on some finite contractible complex. It has been shown that $\gamma_G(F)$ depends only on $\chi(F)$, and so there is an integer $n_G$ such that:

**Theorem 2.** For any group $G$ not of prime power order, a finite complex $F$ is the fixed-point set of an action of $G$ on some finite contractible complex if and only if $\chi(F) \equiv 1 \pmod{n_G}$.

For the calculation of $m(G)$, the following notation will be used. For $q$ prime, let $G^q$ denote the class of all finite groups $G$ with a normal subgroup $H \leq G$ of $q$-power index. Set $G = \bigcup G^q$. Then

**Theorem 3.** If $G \in G^1$, then $m(G) = 0$. If $G \notin G^1$, then $m(G)$ is a product of distinct primes (or 1), and $q | m(G)$ if and only if $G \in G^q$. In particular, $m(G) = 1$ if and only if $G \notin G$.

Attempts to calculate $n_G$ completely have so far been unsuccessful. The best which has been done is to show that $m(G) \mid n_G \mid m(G)^2$. In particular, $n_G = 1$ if and only if $G \notin G$.

Of particular interest is the case of smooth fixed-point free actions on disks. These can be obtained from cellular fixed-point free actions on finite contractible complexes using the methods of [1] or [3]. The above results immediately yield:

**Theorem 4.** A finite group $G$ has a smooth fixed-point free action on a (sufficiently high dimensional) disk if and only if $G \notin G$. In particular, any
nonsolvable group has such an action. A finite abelian group has such an action if and only if it has three or more noncyclic Sylow subgroups.

(Theorem 4 was proved, for solvable groups, in the author’s thesis [3] by different methods.)

REFERENCES


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