inform the reader concerning the historical connection here. At any rate, these Eisenstein series afford another way of constructing automorphic forms on $\mathfrak{H}_n$ with respect to Γ, and are in a certain technical sense complementary to the Poincaré series constructed earlier. Siegel concludes this final chapter with investigation of the field of modular functions (read "automorphic functions with respect to Γ"). He first shows that that *field* at least *is* generated by the Eisenstein series (very little is known about generation of the ring or algebra of modular forms except for Igusa's results for very small n), and then establishes that it is an algebraic function field of finite degree over a purely transcendental extension of $\boldsymbol{C}$ of degree $n(n+1)/2$. The proof of the former fact is Siegel's own (Math. Ann., 1939), and although Siegel established the second fact in the same paper, the proof of it here is due to Andreotti and Grauert, who base their proof on a property of pseudoconcavity connected with the modular group.

Although minor flaws, such as the scarcity of references to the extensive bibliography when there is not space to pursue a subject, and the laxness of the proofreader(s) exist, this reviewer's opinion is that the good far outweighs these trivial shortcomings. These are small blemishes, and the work as a whole stands out for its excellent treatment of a broad subject, and what, for obvious reasons of space, it lacks in completeness, it more than makes up for in inspirational quality.

WALTER L. BAILY, JR.

Algebraic graph theory, by Norman Biggs, Cambridge Tracts in Mathematics No. 67, Cambridge University Press, 1974, vii+170 pp., $11.70

Combinatorial theory seminar, by Jacobus H. van Lint, Eindhoven University of Technology, Lecture Notes in Mathematics No. 382, Springer-Verlag, 1974, vi+131 pp., DM 18

Some of the most satisfying and fruitful developments in mathematics have occurred when bridges have been discovered between seemingly disparate branches of the subject. Then the results and methods of the one branch have become applicable to the other, and at best there has been an equal flow in the reverse direction also. Thus the Zeta function of Riemann allowed complex function theory to illuminate the theory of the distribution of prime numbers, and thereby the theory of entire functions was stimulated also.

Prerequisites, for maximum impact in such situations, are naturalness and depth. That is to say, the bridge should connect problem areas each of which arises naturally in its own subject, several layers beneath the surface. Conversely, little value attaches to an assertion "A is equivalent to B" if A, say, is artificially cooked up for the occasion. Between these two extremes of monumentality and triviality lie a number of intermediate possibilities, of course.

The idea of a spanning tree of a graph arises quite naturally as that of a minimal connected subgraph which visits all vertices. The existence and properties of spanning trees are of importance in graph theory itself and in a variety of applications, such as communications theory. Further, the naturalness of the idea of a determinant in linear algebra needs no amplification here.

Therefore, an event of some importance occurred in 1847 when G. Kirchhoff discovered that the spanning trees of a graph could be counted by a certain determinant (define $a_{ij}=-1$ if i and j are connected by an edge, $=0$ if $i\neq j$ otherwise, $=$the number of edges incident with i, if $i=j$ $(i,j=1,2,\cdots,n)$; calculate any $(n-1)\times(n-1)$ cofactor), and at that moment algebraic graph theory was born. Since that time more and more algebra has been found useful in graph theory, and we are starting to see a reverse flow, of theorems in algebra whose proofs *via* graphs are simple and elegant. Kirchhoff's matrix tree theorem, for example, has been generalized to directed graphs, and has been employed by de Bruijn, Ehrenfest, Smith and Tutte to show (the B.E.S.T. theorem) that the number of Eulerian walks on a directed graph can be counted by a determinant of Kirchhoff's type and a simple multiplicative factor which counts the Eulerian walks associated with each spanning tree. Such counts are, for example, of interest in information-theoretic studies of the DNA molecule!

In the other direction, the Amitsur-Levitskii theorem on polynomial identities in algebras was later shown by Swan to be a consequence of an interesting balance between two kinds of Euler circuits in graphs with enough edges, and this train of thought has been carried forward by B. Kostant, M. Schützenberger, J. Hutchinson, L. Rowen and others.

Chromatic graph theory bristles with the machinery of algebra. Eigenvalues, determinants, simplicial complexes and other constructs have been shown to be related to the coloring of graphs. A judgment as to relevance in this area needs to be suspended until the main problem shows signs of yielding.

The first book under review is an admirable attempt to organize this mass of material. I would take issue with some omissions (the B.E.S.T. theorem, the Amitsur-Levitskii theory), with some emphasis (Kirchhoff is barely mentioned), with some value judgments (Theorem 13.9 is stated to be one of the two most important in the book; the applications which are offered in support of this assertion are first an algorithm which is not analyzed but seems inefficient, and a coefficient inequality which can easily be derived from the familiar delete-and-identify method for chromatic polynomials), and with the number of examples given (there is 1 Figure per 10.9 pages *vs*. 1 per 1.4 pages in Harary's *Graph Theory*, for example). Nonetheless the book gives a good introduction to several active and current research areas such as eigenvalues of graphs and line graphs, characteristic polynomials, coloring problems, the Tutte polynomial, automorphism groups and intersection matrices. It deserves the attention of all serious students of the subject.

In combinatorial mathematics, on the other hand, there is now such vigorous activity on so many fronts that an overview of the whole battlefield would show only smoke: is the permanent function really fundamental or just a gadget? same for the Möbius function; is the four color problem a dead end or is its fallout stimulating the whole subject? in what ways will the blossoming study of computer algorithms reshape combinatorics itself? The issues are, of course, not posed in these stark terms, but instead, all areas are being advanced simultaneously, to the despair of those who seek unifying threads.

The second book under review carefully limits and beautifully achieves its objectives. In 1972–1973 a seminar was held at Eindhoven whose subject was taken to be Marshall Hall's text "Combinatorial Theory". The distinguished mathematicians in attendance (N. G. de Bruijn, M. L. J. Hautus, H. J. L. Kamps, J. H. van Lint, K. A. Post, C. P. J. Schnabel, J. J. Seidel, H. C. A. van Tilborg, J. H. Timmermans, J. A. P. M. van de Wiel) studied Hall's book and wherever they could, extended, amplified, generalized and illustrated the results therein.

This effort has resulted in a number of outstanding contributions, expounded with crystal clarity. Here are to be found a best possible lower bound for systems of distinct representatives (Ostrand, van Lint), a discussion, in the framework of Nijenhuis-Wilf, of upper bounds for permanents of 0–1 matrices, a contour integral solution of the ménages problem (de Bruijn), a generalization of de Bruijn sequences to alphabets of more than two letters (van Lint), new contributions to the theory of Hadamard matrices (Delsarte, Goethals, Seidel), and so forth, encompassing virtually every subject treated by Hall, and done in a precise, luminous style which recommends it to spectators as well as to participants in the arena that is combinatorics today.

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