Let $G$ be a compact Lie group and $N$, $M$ and $Y \subseteq M$ be smooth $G$ manifolds. Suppose $f : N \rightarrow M$ is a proper $G$ map. We give an obstruction theory (Theorem 1) for a proper $G$ homotopy between $f$ and a map $g$ transverse to $Y$ written $f \cap Y$. In this generality we cannot say more; however, when $f : N \rightarrow M$ is a quasi-equivalence of $G$ vector bundles over $Y$, this can be considerably improved (Theorem 2) by removing the dependence of the map $f$. By definition $f$ is a quasi-equivalence if $N$ and $M$ are $G$ vector bundles over $Y$ and $f$ is proper, fiber preserving and degree 1 on fibers. To be concise we suppose $G$ is abelian and omit applications and insights, referring to [1] and [2] for further information.

Let $K$ be a subgroup of $G$ and $\hat{K}$ the set of real irreducible $K$ modules. If $\Gamma$ and $\Omega$ are real $K$ modules, let $V_{\Gamma, \Omega}$ denote the space of surjective real $K$ homomorphisms of $\Gamma$ to $\Omega$. By Schur’s lemma $V_{\Gamma, \Omega} = \Pi_{\psi \in \hat{K}} V_{\Gamma, \Omega}^{\psi}$ where $V_{\Gamma, \Omega}^{\psi}$ has the homotopy type of the Stiefel manifold of $b_{\psi}$ frames in the $D_{\psi}$ vector space of dimension $a_{\psi}$. Here $D_{\psi}$ is the division algebra of real $K$ endomorphisms of $\psi$ and $\Gamma = \Sigma_{\psi \in \hat{K}} a_{\psi} \psi$, $\Omega = \Sigma_{\psi \in \hat{K}} b_{\psi} \psi$.

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Let $L$ denote the set of isotropy groups of the action of $G$ on $N$ partially ordered by inclusion. If $L$ has $T$ elements, choose a 1-1 function $\alpha$ from $L$ to the integers 1 through $T$ with the property that $\alpha(K) < \alpha(H)$ if $K > H$.

Suppose $f$ is transverse to $Y$ on $\bigcup_{\alpha(H) < k} N^H = Z_{k-1}$ and $\alpha(K) = k$. Without loss of generality, we may suppose $f^K \cap Y^K$ and define $X^K = (f^K)^{-1} Y^K$ where $Y^K$ is the fixed set of $K$ acting on $Y$. The $G$ normal bundle of $Y$ in $M$ is denoted by $\nu(Y, M)$. Define $\nu(Y, M)_K$ to be the $G$ complement of $\nu(Y^K, M^K)$ in $\nu(Y, M)|_{Y^K}$. Define a function $V(K)$ from the set of components of $X^K$ to topological spaces whose value at a component $C$ of $X^K$ is $V_{\Gamma, \Omega}$. For $p$ a point in the component $C$ of $X^K$, $\Gamma = \nu(N^K, N)|_p$ and $\Omega = \nu(Y, M)_K|_p$. Set $X_K = \bigcup_{H > K, H \in L} X^K$.

**Theorem 1.** There is a sequence of obstructions $O_*(K) \in H^*(X^K/G, X_K/G, \pi^{*-1}(V(K)))$ (in the sense that $O_j(K)$ is defined if $O_i(K) = 0$ for $i < j$) whose vanishing implies $f$ is properly $G$ homotopic rel $Z_k$ to a function transverse to $Y$ on $Z_k$.

**Theorem 2.** Let $f': N \to M$ be a quasi-equivalence of $G$ vector bundles over $Y$. Suppose $f$ is properly $G$-homotopic to $f'$ and is transverse to $Y$ on $Z_{k-1}$ and $f^K \cap Y^K$. There are obstructions $O'_*(K) \in H^*(Y^K/G, Y_K/G, \pi^{*-1}(V'(K)))$ whose vanishing implies $f$ is properly $G$ homotopic rel $Z_k$ to a function transverse to $Y$ on $Z_k$. (Here $V'(K)$ is a function of the components of $Y^K$.)

**References**


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