CONVERGENCE OF FOURIER SERIES ON
COMPACT LIE GROUPS

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Let $G$ be a compact connected semisimple Lie group. Fix a maximal torus $T$ and denote its Lie algebra by $\mathfrak{T}$. The irreducible unitary representations of $G$ are indexed by a semilattice $L$ of dominant integral forms on $\mathfrak{T}$. For each $\lambda$ in $L$ let $\chi_\lambda$ and $d_\lambda$ be the character and degree of the representation corresponding to $\lambda$.

By the Fourier series of a function $f$ on $G$ we mean the formal series

$$
\sum_{\lambda \in L} d_\lambda \chi_\lambda \ast f.
$$

In this paper we announce results concerning the convergence properties (both mean and pointwise) of polyhedral partial sums of these Fourier series. Details and proofs will appear elsewhere.

Let $P$ be an open, convex polyhedron in $\mathfrak{T}$ centered at the origin. Assume $P$ is Weyl group invariant. Let $RP = \{RX|X \in P\}$ and $S_R f(g) = \sum_{\lambda \in RP} d_\lambda \chi_\lambda \ast f(g)$.

**Theorem A.** If $p \neq 2$ there is an $f$ in $L^p(G)$ such that $S_R f$ does not converge to $f$ in the $L^p$ norm.

An immediate corollary of this theorem is that when $p < 2$ almost everywhere convergence fails for some $f$ in $L^p(G)$. However, the convergence behaviour of Fourier series of functions having invariance properties, in particular class functions, is markedly different.

A class function is a function $f$ such that $f(gxg^{-1}) = f(x)$ for all $g$ in $G$ and almost all $x$ in $G$. Let $L^p_c(G)$ denote the $p$-integrable class functions. For $f$ in $L^p_c(G)$,

$$
d_\lambda \chi_\lambda \ast f(g) = \left( \int f(x) \chi_\lambda(x) dx \right) \chi_\lambda(g).
$$

Let $n = \dim G$ and $l = \text{rank } G = \dim T$.

We now assume that $G$ is a simple, simply connected compact Lie group.

**Theorem B.** If $p > 2n/(n + l)$ and $f$ is in $L^p_f(G)$ then $S_R f(g)$ converges to $f(g)$ for almost all $g$.

**Theorem C.** If $p < 2n/(n + l)$ or $p > 2n/(n - l)$ there is an $f$ in $L^p_f(G)$ such that $S_R f$ does not converge to $f$ in the $L^p$ norm.


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REMARKS. Clerc [1] has proved Theorem A for spherical partial sums. The argument for polyhedral partial sums involves a reduction to the rank one case. If the rank of $G$ is one, Theorem B is due to Pollard [4] while Theorem C was obtained by Wing [6]. For general rank a slightly weaker version of Theorem C was obtained by Stanton [5]. The proofs of our results are extensions of the rank 1 arguments coupled with Fefferman's results [2], [3]. A calculation of the integrability of powers of Weyl's $\Delta$-function and a related function provided the critical indices.

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BIBLIOGRAPHY