A Banach space \( X \) is called primary (resp. prime) if for every projection \( P \) on \( X \), \( PX \) or \((I - P)X\) (resp. \( PX \) with \( \dim PX = \infty \)) is isomorphic to \( X \). It is well known that \( c_0 \) and \( l_p \), \( 1 < p < \infty \), are prime spaces \([5], [8]\), but it is an open question whether there are other prime Banach spaces. However, it is known that \( C[0, 1] \) \([7]\) and \( L_p[0, 1], 1 < p < \infty \) \([1]\), are primary, and in the recent Special Seminar on Functional Analysis at Urbana, Illinois, August, 1975, it is announced \([2]\) that \( C(K) \) is primary for any countable compact metric space \( K \). For a discussion on prime and primary Banach spaces, we refer to \([6]\).

For a Banach sequence space \((E, \|\cdot\|_E)\) and a sequence of Banach spaces \(\{X_n\}_n\), we shall let \((X_1 \oplus X_2 \oplus \cdots)_E\) be the Banach space of all sequences \(\{x_n\}_n\) such that \(x_n \in X_n, n = 1, 2, \ldots\) and \((\|x_1\|, \|x_2\|, \ldots) \in E\) with the norm \(\|(x_n)\| = \|(x_1, x_2, \ldots)\|_E\).

A basis \(\{e_n\}_n\) in a Banach space \(X\) is called symmetric (cf. \([10]\)) if every permutation \(\{e_{\pi(n)}\}_n\) of \(\{e_n\}_n\) is a basis of \(X\), equivalent to \(\{e_n\}_n\). For a basis \(\{e_n\}_n\) of a Banach space \(X\), we shall let \(X_n\) be the linear span of \(e_1, e_2, \ldots, e_n\) in \(X\).

**MAIN THEOREM.** Let \(X\) be a Banach space with symmetric basis \(\{e_n\}_n\). Then the following spaces are primary.

(i) \((X_1 \oplus X_2 \oplus \cdots)_E\), \(1 < p < \infty\), where \(X\) is not isomorphic to \(l_1\).

(ii) \((X_1 \oplus X_2 \oplus \cdots)_E\), \(1 < p < \infty\), and \((X_1 \oplus X_2 \oplus \cdots)_c\).

(iii) \((l_\infty \oplus l_\infty \oplus \cdots)_E\), \(1 < p < \infty\), and \((l_\infty \oplus l_\infty \oplus \cdots)_c\).

Different techniques are needed in each of the three cases, and the cases \(p = 1\) or when \(X\) is isomorphic to \(l_1\) have to be treated separately. The proof for (i) is similar to the technique developed in \([3]\). To prove (ii), we use Ramsey’s combinatorial lemma \([9]\) and the following

**LEMMA.** Let \(M = \{m_i\}_i\) be a sequence of positive integers such that \(\lim \sup m_i = \infty\). Then there exist rearrangements of \(M\) and the set of positive integers \(N\) into double sequences \(\{m'_1, m'_2, \ldots; m''_1, m''_2, \ldots\}\) and \(\{n'_1, n'_2, \ldots; n''_1, n''_2, \ldots\}\) such that \(m'_i = n'_{2i-1} + n'_{2i}\) and \(m''_i = n''_{2i-1} + m''_{2i}\) for \(i = 1, 2, \ldots\).
**Corollary.** Let $X$ be a Banach space with symmetric basis and let \( \{B_n\} \) be a sequence of Banach spaces with $\dim B_n = n$, $n = 1, 2, \ldots$. If there exists a constant $K$ such that the Banach-Mazur distance

\[
\overline{d}(B_n \oplus B_m, B_{n+m}) = \inf \{ \| T \| \| T^{-1} \| : T: B_n \oplus B_m \to B_{n+m} \text{ linear isomorphism} \} \leq K
\]

for all $n, m = 1, 2, \ldots$, then $(B_1 \oplus B_2 \oplus \cdots)_X$ is isomorphic to $(B_{m_1} \oplus B_{m_2} \oplus \cdots)_X$ for all $\{m_i\}$ with $\lim \sup m_i = \infty$.

**Remark.** When $X = l_p$, $1 < p < \infty$, a similar result was stated in [4, Lemma 5].

The proof of (iii) consists of generalizing the technique used by Lindenstrauss [5] in proving that $l_\infty$ is prime and the following fact which is interesting in itself.

**Theorem 2.** Let $X$ be a Banach space with symmetric basis. If $E$ is a Banach space which has a complemented subspace isomorphic to $X$, then for any bounded linear operator $T: E \to E$, either $TE$ or $(I - T)E$ has a complemented subspace isomorphic to $X$.

By combining the techniques used to prove the Main Theorem and Theorem 2, we could obtain, for example,

**Theorem 3.** Let $X$ be a Banach space with symmetric basis. If $E$ is a Banach space which has a complemented subspace isomorphic to $(X \oplus X \oplus \cdots)_p$, $1 < p < \infty$ (resp. $(X \oplus X \oplus \cdots)_{c_0}$), then for any bounded linear operator $T: E \to E$, either $TE$ or $(I - T)E$ contains a complemented subspace isomorphic to $(X \oplus X \oplus \cdots)_p$, $1 < p < \infty$ (resp. $(X \oplus X \oplus \cdots)_{c_0}$).

Details of proofs will appear elsewhere.

**References**

2. Dale Alspach and Y. Benjamini, *On the primariness of $C(K)$ where $K$ is a countable compact metric space*, in preparation.


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