Abstract. We give generalizations and extensions concerning integrability of shifted and weighted trigonometric series of Boas and Rees-Stanojević.

Boas [1] proved the following two integrability theorems for certain weighted sine and cosine series.

**Theorem A.** Let \( g(x) = \sum_{n=1}^{\infty} b(n) \sin nx \) where \( b(n) \) decreases to zero. Then for \( 0 \leq \gamma < 1 \), \( x^{-\gamma}g(x) \in L[0, \pi] \) if and only if \( \sum_{n=1}^{\infty} n^{\gamma-1} b(n) < \infty \).

**Theorem B.** Let \( f(x) = \sum_{n=1}^{\infty} a(n) \cos nx \) where \( a(n) \) decreases to zero. Then for \( 0 < \gamma < 1 \), \( x^{-\gamma}f(x) \in L[0, \pi] \) if and only if \( \sum_{n=1}^{\infty} n^{\gamma-1} a(n) < \infty \).

Recently Rees and Stanojević [2] proved a similar theorem for a shifted sine series.

**Theorem C.** Let \( g(x) = \sum_{n=1}^{\infty} b(n) \sin(n + \alpha) x \) where \( b(n) \) decreases to zero. Then \( x^{-1}g(x) \in L[0, \pi] \) if and only if \( \sum_{n=1}^{\infty} b(n) < \infty \).

Theorem C is a by-product of an integrability theorem for certain cosine sums introduced in [2]. It follows after summation by parts of these cosine sums due to the form of the Dirichlet kernel. This paper gives extensions of Theorems A and B in the direction indicated by Theorem C.

**Theorem 1.** Let \( g(x) = \sum_{n=1}^{\infty} b(n) \sin(n + \alpha) x \) where \( b(n) \) decreases to zero and \( 0 \leq \alpha \leq \frac{\pi}{2} \). Then for \( 0 \leq \gamma < 1 \), \( x^{-\gamma}g(x) \in L[0, \pi] \) if and only if \( \sum_{n=1}^{\infty} n^{\gamma-1} b(n) < \infty \).

**Theorem 2.** Let \( f(x) = \sum_{n=1}^{\infty} a(n) \cos(n + \alpha) x \) where \( a(n) \) decreases to zero and \( 0 \leq \alpha \leq \frac{\pi}{2} \). Then for \( 0 < \gamma < 1 \), \( x^{-\gamma}f(x) \in L[0, \pi] \) if and only if \( \sum_{n=1}^{\infty} n^{\gamma-1} a(n) < \infty \).

Proofs and details of these theorems will appear elsewhere.

REFERENCES


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