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AUGMENTED TEICHMÜLLER SPACES

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The augmented Teichmüller space $\hat{T}$, of a finitely generated Fuchsian group G of the first kind or a conformally finite Riemann surface S with signature, consists of the usual Teichmüller space T together with the regular b-groups on its boundary. The structure of the regular b-groups has been studied in [2] (see also Marden [5] and Maskit [6]). The usual topology on T given by the Bers embedding of T in the space of bounded quadratic differentials has a natural extension to $\hat{T}$. The extension corresponds to horocycles at the regular b-groups. It is discussed in §2. Some of the properties of $\hat{T}$ with this topology are listed below. Detailed proofs will appear elsewhere. A related study is being conducted by Earle and Marden.

1. Properties of $\hat{T}$.

THEOREM 1. *Each element g of the Teichmüller modular group,* Mod, *has a continuous extension to an automorphism of* $\hat{T}$.

The proof of Theorem 1 follows from explicit construction of quasiconformal mappings realizing twist maps and transpositions.

THEOREM 2. *The augmented Riemann space* $\hat{R} = \hat{T}/\text{Mod}$ *is a compact normal complex space. It is the unique compactification of* $R = T/\text{Mod}$ *in the sense of Cartan.*

The proof utilizes a correspondence between congruence classes of regular b-groups and flags of subgroups of Mod. The uniqueness of the compactification together with results due to Bers [3] immediately yield

THEOREM 3. $\hat{R}$ *is a projective algebraic variety.*

By studying divergent sequences in $\hat{T}$, we may prove the following conjecture of Ehrenpreis [4].

THEOREM 4. *If T is given some Bers embedding, then the action of* Mod *is of the first kind (i.e. for each* $\varphi \in \partial T$*, each Euclidean neighborhood* N *of* φ *and each n, there is some* $\varphi_1 \in N \cap T$ *whose orbit meets* N *in at least n points).*

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We also study sheaves of q-differentials over $\hat{T}$, and their relationship to the Poincaré Θ-operator and Bers' L-operator. The normalizations required, hence the results, are too complicated to state here.

2. **The topology of $\hat{T}$.** We utilize Bers' notion of Riemann surface with nodes, with the more or less obvious extension to marked surfaces S with nodes and signature (see [1] and [2]). A deformation $\langle S_1, S_2, f\rangle$ consists of a surjection f: $S_1 \longrightarrow S_2$ with the following properties:

(i) f^{-1}(node) is either a node, a simple loop or a slit connecting two ramification points of order two.

(ii) $f^{-1} \mid (S_2$ nodes) is a local homeomorphism respecting the markings.

Let K be a neighborhood of the nodes on S_2. A (K, ϵ)-$\mathcal{C}$-neighborhood of S_2 is the set of marked surfaces S with nodes and signature such that a deformation $\langle S, S_1, f\rangle$ may be chosen with $f^{-1} \mid (S - K)$ $(1 + \epsilon)$-quasiconformal. Similarly, we define a (K, ϵ) $\mathcal{I}$-neighborhood of $f^{-1} \mid (S - K)$ is $(1 + \epsilon)$-quasi-isometric. These define the same topology on $\hat{T}$. The topology coincides with that defined by lengths of curves converging.

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