AUGMENTED TEICHMÜLLER SPACES

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The augmented Teichmüller space \( \hat{T} \), of a finitely generated Fuchsian group \( G \) of the first kind or a conformally finite Riemann surface \( S \) with signature, consists of the usual Teichmüller space \( T \) together with the regular \( b \)-groups on its boundary. The structure of the regular \( b \)-groups has been studied in [2] (see also Marden [5] and Maskit [6]). The usual topology on \( T \) given by the Bers embedding of \( T \) in the space of bounded quadratic differentials has a natural extension to \( \hat{T} \). The extension corresponds to horocycles at the regular \( b \)-groups. It is discussed in §2. Some of the properties of \( \hat{T} \) with this topology are listed below. Detailed proofs will appear elsewhere. A related study is being conducted by Earle and Marden.

1. Properties of \( \hat{T} \).

THEOREM 1. Each element \( g \) of the Teichmüller modular group, \( \text{Mod} \), has a continuous extension to an automorphism of \( \hat{T} \).

The proof of Theorem 1 follows from explicit construction of quasiconformal mappings realizing twist maps and transpositions.

THEOREM 2. The augmented Riemann space \( \hat{R} = \hat{T}/\text{Mod} \) is a compact normal complex space. It is the unique compactification of \( R = T/\text{Mod} \) in the sense of Cartan.

The proof utilizes a correspondence between congruence classes of regular \( b \)-groups and flags of subgroups of \( \text{Mod} \). The uniqueness of the compactification together with results due to Bers [3] immediately yield

THEOREM 3. \( \hat{R} \) is a projective algebraic variety.

By studying divergent sequences in \( \hat{T} \), we may prove the following conjecture of Ehrenpreis [4].

THEOREM 4. If \( T \) is given some Bers embedding, then the action of \( \text{Mod} \) is of the first kind (i.e. for each \( \varphi \in \delta T \), each Euclidean neighborhood \( N \) of \( \varphi \) and each \( n \), there is some \( \varphi_1 \in N \cap T \) whose orbit meets \( N \) in at least \( n \) points).
We also study sheaves of \( q \)-differentials over \( \hat{T} \), and their relationship to the Poincaré \( \Theta \)-operator and Bers’ \( L \)-operator. The normalizations required, hence the results, are too complicated to state here.

2. The topology of \( \hat{T} \). We utilize Bers’ notion of Riemann surface with nodes, with the more or less obvious extension to marked surfaces \( S \) with nodes and signature (see [1] and [2]). A deformation \( \langle S_1, S_2, f \rangle \) consists of a surjection \( f: S_1 \to S_2 \) with the following properties:

(i) \( f^{-1}(\text{node}) \) is either a node, a simple loop or a slit connecting two ramification points of order two.

(ii) \( f^{-1}(\text{nodes}) \) is a local homeomorphism respecting the markings.

Let \( K \) be a neighborhood of the nodes on \( S_2 \). A \((K, \epsilon)\)-\( C \)-neighborhood of \( S_2 \) is the set of marked surfaces \( S \) with nodes and signature such that a deformation \( \langle S, S_1, f \rangle \) may be chosen with \( f^{-1}(S - K) (1 + \epsilon) \)-quasiconformal. Similarly, we define a \((K, \epsilon)\)-\( I \)-neighborhood of \( f^{-1}(S - K) \) is \((1 + \epsilon)\)-quasi-isometric. These define the same topology on \( \hat{T} \). The topology coincides with that defined by lengths of curves converging.

REFERENCES


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