mathematician who already knows quantum mechanics will likely be much more excited by a book such as D. B. Lichtenberg's *Unitary symmetry and elementary particles.* This book, incidentally, contains a lovely explanation of the relationships among conservation laws, symmetries, and group representations. These points are virtually untouched by van der Waerden. For that matter, I might mention that $SU(n)$ for $n \geqq 3$ and, in particular, $SU(3)$, groups of considerable importance in modern physics, are totally absent from van der Waerden's book.

For the mathematician who would like to learn about quantum mechanics for the first time, I would recommend the Feynman *Lectures on physics.* As is well known, the third volume of these lectures achieves a remarkable tour de force: quantum mechanics is presented with only the most elementary use of mathematics. It would be nice to have, in addition, a book complementary to Feynman's a book which assumes considerable mathematical knowledge and maturity and yet which presents quantum mechanics with a minimum of physical prerequisities, but with the same force and sense of reality as in the Feynman lectures.

ALAN HOPENWASSER

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Jean Dieudonné,[1] *Cours de géométrie algébrique.* Vol. 1: *Aperçu historique sur le développement de la géométrie algébrique*, 234 pp.; Vol. 2: *Précis de géométrie algébrique élémentaire*, 222 pp., Collection SUP, Presses Universitaires de France, Paris, 1974, paperback, pocketbook size. Each vol. 34.88F in France = approx. $8.30.

I. R. Shafarevich,[2] *Basic algebraic geometry*, Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 213, Springer-Verlag, Berlin, 1974, xv+439 pp., $40.20. (Translated from the Russian by K. A. Hirsch)

The author of an introductory book on algebraic geometry faces many difficult choices. How is he to introduce his reader to some of the basic examples of algebraic geometry, give him some motivation, and teach him the modern language of the subject? As Dieudonné says in his introduction, "Algebraic geometry is surely that branch of mathematics having the greatest gap between the intuitive ideas which form the point of departure and the complex abstract concepts which lie at the base of modern research." No introductory book will succeed unless it makes a serious attempt to bridge this gap.

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[1] Dieudonné's Volume 1 is an expanded version of an article *The historical development of algebraic geometry*, Amer. Math. Monthly **79** (1972), 827–866.

[2] The first four chapters of Shafarevich's book are almost identical with an earlier book, which has appeared in English translation as *Foundations of algebraic geometry*, Russian Math. Surveys **24** (1969), 1–178.

The first choice facing our author is the choice of language. Algebraic geometry has its roots firmly in the 19th century, with the function-theoretic approach of Riemann, the more geometric approach of Brill and Max Noether, and the algebraic foundations given by Kronecker, Dedekind and Weber. Since then it has developed in stages. There is the Italian school of Castelnuovo, Enriques, and Severi, culminating in the classification of algebraic surfaces. There is the "American" school of Chow, Weil and Zariski which gave rigorous algebraic foundations (especially as regards the treatment of singularities) to the Italian intuition. The most recent revolution has been the introduction of sheaf cohomology and schemes by Serre and Grothendieck. This last approach has led to a vast program of rewriting the foundations of algebraic geometry, and has also had an impressive record of solving old problems with its new methods. Each new school has brought new concepts and new terminology. Should one use older language which is closer to the geometric intuition, or should one begin right away with the abstract technical language used in current research papers?

The second choice facing our author is a conceptual one. While the modern student need not necessarily be familiar with all phases of the historical development of the subject, he should certainly have a good experience of the basic objects of study, which are algebraic varieties in affine or projective space, and he should have some idea of the input to algebraic geometry coming from complex manifolds, number theory, and commutative algebra. On the other hand, to be an educated algebraic geometer in the 1970's, he must be familiar with the technical apparatus of schemes and cohomology. How can the author of an introductory book create an effective balance between basic examples, motivation from other fields, and the formal development of the subject in modern language? What choice of topics will give a real sense of what algebraic geometry is about, and at the same time serve as a firm foundation for further study and research?

Now let us see how our two authors have faced up to this rather formidable challenge.

Dieudonné's solution is to write two little books. The first is a historical account of the main ideas and results in algebraic geometry from 400 B.C. to 1973 A.D. The second is a self-contained development of the foundations of algebraic geometry, written in the language of "Serre varieties", which are essentially reduced schemes of finite type over an algebraically closed ground field. His hope is that the first volume will given an idea of what algebraic geometry is about, and that the second will constitute the first step towards a serious study of the subject.

I would say that the first volume is a complete success. Written in a beautiful clear style, it contains complete (if hasty) definitions of all the concepts involved, traces all the major ways of thinking, and gives statements of results obtained, ranging from the projective geometry of Poncelet (ca. 1810) to the resolution of singularities of Hironaka (1964), and from the transcendental methods of Kähler varieties and Hodge theory to the zeta functions of varieties over finite fields. The last chapter has a selection of recent results and open problems. It is a book which can be profitably read and reread by beginner and expert alike. My only quibble is that the references are often not sufficient to locate the results quoted in the literature. For example there is no reference for Chasles' calculation (p. 40) of the number of conics tangent to 5 given conics; nor is there a reference for Castelnuovo's determination (p. 84) of the maximum genus of a space curve of degree d.

The second volume, on the other hand, is of much more limited interest. Coming from the coauthor of the great treatise *Eléments de géométrie algébrique* (EGA), one has the impression that this is a book of watered-down schemes. To be sure, great technical simplifications over the general theory of schemes are achieved by restricting one's attention to Serre varieties. But, like EGA, this book is unsuitable for beginners because there are not enough examples. The reader is never told *why* he should be interested in certain concepts. And if the reader is not a beginner, he can spend his time more profitably reading something else. Also I believe the choice of topics is unsuitable for this level. To introduce abstract varieties before having a good experience of projective varieties is inappropriate. To spend ten pages on subtle variations of Zariski's Main Theorem before one has studied the classification of surfaces is absurd. The one part which may be useful is §4 on dimension theory, since it gives the theory of constructible sets and the dimension of the fibres of a morphism, which is not easily accessible in the literature.

In spite of these criticisms, one has to admire the tight structure of the book. The necessary preliminaries of commutative algebra and topology are gathered into two appendices, with proofs whenever a reference to Lang's *Algebra* or Bourbaki's *Topologie générale*, Chapter I, would not suffice. In this way the book becomes self-contained. In comparison with Shafarevich's book, one is tempted to draw a parallel to Molière and Tolstoy. In the plays of Molière one has unity of time, place and action. By restricting one's attention to a small arena, one can achieve a satisfying sense of perfection, balance, and completeness. The novels of Tolstoy on the other hand paint a broad picture of life in all its aspects, always exploring connections with the outside. I have a similar feeling in comparing these two books.

Shafarevich's response to the challenge of writing an introduction to algebraic geometry is a long book, divided into three parts. The first part (220 pp.) deals with algebraic varieties in projective space over a field. The second part (80 pp.) is an introduction to schemes (without cohomology), and the third part (100 pp.) is a study of topological and analytic properties of algebraic varieties over the complex numbers. There is a brief historical sketch (20 pp.) at the end.

The first part is an excellent introduction to the basic properties and concepts associated with projective varieties. In contrast to the French tendency of writing pages and pages of generalities before giving any real examples, Shafarevich always proceeds from the particular to the general.

Thus already in the first six pages of his text, he has defined the notion of a rational curve, and has proved that a certain plane cubic curve is not rational. This is a rather subtle question for one's very first contact with algebraic geometry, but at least there is no doubt that one is dealing with serious issues! In this manner, with gradually increasing generality, he develops the basic notions of affine and projective varieties, dimension, normal and simple points, blowing-up, divisors, differential forms, intersection theory, culminating with the theorem that any birational transformation of nonsingular surfaces can be factored into a finite sequence of blowing-ups. The text is accompanied by copious examples and exercises, so that a reader who does them all will have an excellent experience of actual examples to bolster his intuition.

While the development of the text is elementary, Shafarevich always has his eyes open and alludes when appropriate to more advanced material and occasional unsolved problems. Some results, such as the Riemann-Roch theorem for curves, are included without proof, and there is a nice section of 8 pages giving the basic facts about algebraic groups, also without proofs.

Results of a purely algebraic nature are proved as needed throughout the text, usually by reducing to questions about polynomial rings. While this has the advantage of keeping the algebra elementary, I think it would often be clearer to refer to some deeper results of commutative algebra. For example there is a direct proof (p. 94) that the local ring of a simple point is a unique factorization domain, but there is no mention of the general facts that the local ring of a simple point is a regular local ring, and that every regular local ring is a UFD.

All in all, this first part of the book is an excellent first introduction to algebraic geometry, since it gives a good experience of the basic objects of study in not too fancy language.

The second part of the book is a minimal introduction to schemes, coherent sheaves, and, as a special case, abstract varieties. While giving a first taste of schemes, it will not be sufficient for the serious student, because it does not go far enough. There is only one real application of the new methods, which is to prove the finiteness of the vector space $L(D)$ of rational functions on a variety whose poles are bounded by a given divisor D. This is proved as a special case of the finiteness of the vector space of global sections of a coherent sheaf.

The third part of the book, concerning algebraic varieties over the complex numbers, does a real service in that it records many results of "folklore" which are difficult to locate in the literature, and does this with a minimum of prerequisites from complex analysis and algebraic topology. Here we find the topological structure of an algebraic curve as a compact oriented 2-manifold; a comparison of the category of nonsingular algebraic varieties over $\mathbf{C}$ with the category of complex manifolds, with examples; and a brief account of an algebraic curve as a quotient of a simply connected Riemann surface by a discrete group of automorphisms. This part of the book will be especially useful to the reader already familiar with complex manifolds.

There are a number of typographical errors in Shafarevich's book. In particular, many of the bibliographical references are misnumbered. There is one mathematical error: in the definition of a scheme (p. 244) one must consider *locally* ringed spaces, and require that all morphisms induce *local* homomorphisms of the stalks.

In conclusion, we can say that Dieudonné's history should be read by everyone interested in algebraic geometry, and that Shafarevich's book—at least until the publication of some other introductory algebraic geometry texts now in preparation—is a serious contender for "the best modern introduction to algebraic geometry".

ROBIN HARTSHORNE

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Foundations of special relativity: *Kinematic axioms for Minkowski space-time*, by J. W. Schutz, Lecture Notes in Mathematics, No. 361, Springer-Verlag, Berlin, Heidelberg, New York, 1973, xx+314 pp.

Since the latter part of the nineteenth century, the papers on the axiomatic foundations of Euclidean geometry and the closely related projective, affine, hyperbolic, and elliptic geometries are to be numbered in the thousands. The absence of a comparable flow of papers about the axiomatic foundations of special relativity is hard to understand, especially in view of the fact that Einstein's basic theoretical paper on special relativity appeared in 1905 [**2**], which is close to the heyday of foundational studies of Euclidean geometry.

During the early part of this century, almost the only person doing any work on the qualitative foundations of special relativity was Alfred A. Robb, who first began publishing on the subject in 1911, followed by a small book in 1913, a revision of that book in 1921, and a full-scale work in 1936 [**5**]. Robb's axiomatization of the geometry of special relativity is important for several reasons. First of all, he uses an extremely simple single primitive concept, the binary relation of one space-time event's being *after* another. This is a simpler primitive in logical structure than any of those that have been used for the foundations of Euclidean geometry, and for good reason. Tarski showed many years ago that no nontrivial binary relation can be defined in Euclidean geometry and consequently there is no hope of basing Euclidean axioms on a binary relation between points.

On the other hand, the complexity of Robb's axioms stands in marked contrast to the simplicity of the single primitive concept. If I cited the full set of axioms here, the reader would be appalled by their length and, in many cases, relative difficulty of intuitive comprehension.

Shortly after World War II, A. G. Walker in several publications [**9**], [**10**] offered a new qualitative foundation of the geometry of special relativity. In addition to the set of space-time events, he used particles, an ordering relation of beforeness on events, and, perhaps most importantly, a one-one