$$\exp(xz + tz^2) = \sum_{n=0}^{\infty} v_n(x, t)z^n/n!;$$

other series of polynomials are also useful, just as series of polynomials other than $\{z^n\}$ are important in complex analysis. Temperature functions possess a maximum principle, a reflection principle, and uniqueness theorems showing how they are determined by various kinds of data; there is even an analogue of Liouville's theorem. For a suitably restricted subclass of temperature functions there is Huygens' principle (which gets its name from a quite different analogous theory, optics, i.e. the theory of the wave equation); this says that the values of the function for some t can be used as initial data for determining the function at later values of t, in much the same way that we can take the values of an analytic function on a contour and use them in Cauchy's formula to calculate the function inside the contour. (Poisson's formula for harmonic functions is perhaps a closer analogue.) The analogy between positive temperature functions and positive harmonic functions has already been mentioned. One chapter is devoted to the use of Jacobian theta functions for solving the heat equation in a finite x-interval; the occurrence of these functions is less surprising than one might think, since the theta functions are series of functions $k(x, t)$ or $k_x(x, t)$; they also occur in the construction of the Green's function for an (x, t)-rectangle. One chapter indicates some possible generalizations to higher dimensions; another discusses homogeneous temperature functions ($u(\lambda x, \lambda^2 t) = \lambda^n u(x, t)$). A final chapter considers several special topics.

The book is written in the author's customary polished but condensed style. Much of it consists of simplified versions of his own previous work. The results seem, generally speaking, to be more difficult than their analogues in complex analysis; I do not know whether this is because the latter theory is longer established or because problems about the heat equation are inherently more difficult than problems for Laplace's equation (as suggested to me by A. Friedman). It seems likely, however, that many additional interesting results are waiting to be discovered (or invented, depending on our philosophy of mathematics). Anyone who wishes to participate in the search should have this book at hand.

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Continuous flows in the plane, by Anatole Beck, Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Band 201, Springer-Verlag, New York, Heidelberg, Berlin, 1974, x + 462 pp., $46.80.

A *flow* in a space X is a (continuous) group action of the real line on X; that is, a continuous function φ: $\mathbf{R} \times X \to X$ such that $\varphi(t + s, x) = \varphi(t, \varphi(s, x))$. Behind this simple analytic veil lies, in the case where X is the plane $\mathbf{R}^2$ (or the two-sphere S^2), a beautiful geometric theory. The plane becomes a patchwork quilt. The patches come in infinitely varied and intriguing patterns, that, nevertheless, admit to a surprising amount of classification.

The goal is to disassemble the quilt, describe the individual patches, and sew them back together the way they were. This approcach is geometric, topological–in contrast to the classical analytic approach, with its less pervasive geometric flavor.

There are easy patches and there are tough patches, and there are patches within patches, even a telescoping infinity of patches within patches. One takes on the easy patches, then does the best one can with the tough ones, utilizing the information obtained about the easy ones–and then some–and leaves the rest of the job to posterity.

One can, of course, start this program with the simplest nontrivial manifold–the line itself. While there is more to the question than immediately comes to mind, the topological-geometric approach soon exhausts all possibilites. The next step is the plane, a natural habitat for flows in any event. The two sphere, in this program, is essentially the same as the plane.

One begins by examining *orbits* $\mathcal{O}(x) = \{\varphi(s, x): s \in \mathbf{R}\}$. There are three kinds: (1) one-one images of $\mathbf{R}$, (2) circles (*periodic* orbits), and (3) points (*fixed points* of φ). Fixed points and periodic orbits are closed, but the other kind may not be, so one looks at $\overline{\mathcal{O}(x)} = \mathcal{O}(x) \cup \alpha_\varphi(x) \cup \omega_\varphi(x)$, where $\alpha_\varphi(x) = \bigcap_{t\in\mathbf{R}}\{\varphi(s, x): t \leqslant s\}^-$ and $\omega_\varphi(x) = \bigcap_{t\in\mathbf{R}}\{\varphi(s, x): s \leqslant t\}^-$, the sets of α- and ω-*endpoints* of $\mathcal{O}(x)$, respectively.

To see what these endpoints might be, one tries to trap a tail of $\mathcal{O}(x)$ in a bounded region of the plane. This is done by the important and very useful *Gate Theorem*. (Actually, there are two Gate Theorems, together requiring two pages to state.) The main idea is to find an orbit $\mathcal{O}(x)$ and an arc L_0 joining two points $\varphi(t_1, x) = x_1$, $\varphi(t_2, x) = x_2$ on $\mathcal{O}(x)$ such that the compact region bounded by the Jordan curve $J = (\varphi[t_1, t_2], x) \cup L_0$ can be "entered" by an orbit only through the "gate" L_0, and once entering cannot turn back (the flow has direction–orbits do not go against the current). Thus, by the Jordan Curve Theorem, one end of the orbit is trapped in a bounded region of the plane, and so has compact closure. The set of α- (or ω-) *endpoints* so obtained is therefore compact. Such endpoints can be circular orbits, in which case the geometric pattern is, to a large extent, discernible, or they can be fixed points which are called, under such circumstances, *stagnation points*, where the theory becomes both more complicated and less comprehensive. For flows in the plane, here lies the frontier.

There are, as one can readily see, other types of fixed points. For example, the covering group of the rotation group in the plane has precisely one fixed point, and it is not an endpoint of any orbit. Such fixed points are called *regular* fixed points. This distinction between types of fixed points is probably the most important step in opening the "gate" to the mysteries of flows in the plane. Previously, in the study of flows, a fixed point was a fixed point was a singularity, and was obnoxious. But really the only obnoxious fixed points are stagnation points.

As with fixed points, there are *regular* and *singular* moving points. A periodic orbit is *singular* if it has points arbitrarily close that have stagnation points. Those which are not singular, are *regular*. The regular moving points are the union of disjoint annuli; the singular moving points are what is left of the moving points. The description of the flow then is given in terms of the regular moving points, where it can be completely described in terms of simple "patches"–certain standard annular flows; the fixed points–where it is, of course, fixed; and the singular moving points–where life gets complicated.

To develop the theory for singular moving points, the author first considers a very special case: the case of flows in a multiply-connected region of the plane where every orbit is aperiodic and has all its endpoints in the boundary. These he calls *Kaplan-Markus* flows, after the two who first began development of the theory of such flows. Complete success in describing such flows has eluded the author and his coworkers except, to some extent, where the set of singular fixed points has only finitely many components. The later is, however, basic, and so for many cases, another patch in the quilt yields to description. After this, the author explores various ways to combine such flows with regular flows and develop further theory.

Flows without stagnation points can be described rather completely. For flows with a finite number of stagnation points, or whose set of stagnation points has countable closure, a considerable amount is known, though the information is not as complete as for the no stagnation point case. In all cases where a description is possible, it is the set of regular moving points that supplies the main body of information. However, the author and his coworkers have developed a great deal of information about the singular moving points, forming therewith *organs* of the flow, which in turn are decomposed into *tissues* and *gametes*, and these in their turn are decomposed into *cells*. These "cells," "gametes," and "tissues" are pretty well characterized, and even the "organs" are subject to a good deal of description.

Some additional concepts have been studied, such as the algebra of flows introduced by J. and M. Lewin. One could say that, as of 1975, it is the *complete book about flows in the plane*. It is accessible to anyone with a minimal background in analysis and point set topology–provided one sticks to it sufficiently to keep track of all the terminology and notation peculiar to the book. It is light reading (except for that)–yet builds a substantial theory. It is well written and enjoyable.

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Gaussian measures in Banach spaces, by Hui-Hsiung Kuo, Lecture Notes in Math., no. 463, Springer-Verlag, Berlin, Heidelberg, New York, 1975, vi + 224 pp., \$9.90.

There are difficulties in constructing measures on infinite dimensional spaces. Even in a separable infinite dimensional Hilbert space the unit ball is not compact. Therefore a countably additive measure on such a space cannot