In this note we give a short survey on joint work with K. Behnke; details will appear in [1] and [2].

Let \( n, q \) be positive integers with \( 2 < q < n \) and \( \gcd(n, q) = 1 \), \( m = n - q \). We define elements \( \phi_m, \psi_q, \eta \in \text{GL}(2, \mathbb{C}) \) by

\[
\phi_m = \begin{pmatrix} \xi_{2m} & 0 \\ 0 & \xi_{2m} \end{pmatrix}, \quad \psi_q = \begin{pmatrix} \xi_{2q} & 0 \\ 0 & \xi_{2q}^{-1} \end{pmatrix}, \quad \eta = \begin{pmatrix} 0 & \iota \\ \iota & 0 \end{pmatrix},
\]

where \( \iota = \sqrt{-1} \) and \( \xi_k = \exp(2\pi i/k) \). The group \( G_{n,q} \subset \text{GL}(2, \mathbb{C}) \) is generated by

(a) \( \phi_m, \psi_q, \eta \) in case \( m \) odd,
(b) \( \psi_q, \eta \circ \phi_{2m} \) in case \( m \) even.

\( G_{n,q} \) has finite order \( 4mq \); \( G_{q+1,q} \) is the binary dihedral group of order \( 4q \).

\( G_{n,q} \) acts on \( \mathbb{C}^2 \) in the usual way; the quotient \( \mathbb{C}^2/G_{n,q} \) has precisely one (normal) complex-analytic singularity. We call it the dihedral singularity of type \( D_{n,q} \). If we expand \( n/q \) into the modified continued fraction à la Hirzebruch-Jung,

\[
n/q = b_3 - \frac{1}{b_4 - \cdots - \frac{1}{b_r}}, \quad b_r \geq 2, \quad r \geq 4,
\]

it can be characterized by the dual graph of its minimal resolution (cf. [3]):

\[
\begin{array}{c}
\bullet \quad \bullet \quad \bullet \quad \bullet \\
-2 \quad -b_3 \quad -b_4 \quad -b_r \\
\end{array}
\]

\( \cong \mathbb{P}_1(\mathbb{C}) \).

The equations are calculated by invariant theory. As in the cyclic group case [5], we put

\[
n/m = a_2 - \frac{1}{a_3 - \cdots - \frac{1}{a_{e-1}}}, \quad a_e \geq 2.
\]

Further set \( A_3 = a_3 + 1, \quad A_e = a_e, \quad e \neq 3, \) and

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Then we have

**Theorem 1.** A minimal set of generators for $S_{n,q} = C[u,v]^G$ is formed by the polynomials

$$z_1 = (uv)^2m, \quad z_e = (uv)^r_e(u^{2qse} + (-1)^e v^{2qse}), \quad e = 2, \ldots, e.$$  

After a (noncanonical) change of variables it is possible to find simple equations.

**Theorem 2.** The dihedral singularity of type $D_{n,q}$ is (minimally) described by $\frac{1}{2}(e - 1)(e - 2)$ equations

$$z_2^2 = z_1(z_3^2 + z_1^{a_2-1}),$$

$$z_1 z_e^{2} = z_2 z_3^{a_3-2} \cdots z_e^{a_e-2} z_{e-1}^{a_{e-1}-1}, \quad e = 4, \ldots, e,$$

$$z_2 z_e = z_3^{a_3-1} \cdots z_e^{a_{e-1}-1}(z_3^2 + z_1^{a_2-1}), \quad e = 4, \ldots, e,$$

$$z_{e-1} z_{e+1} = z_e^a, \quad e = 4, \ldots, e - 1,$$

$$z_2 z_e = z_3^{a_3+1-1} z_5^{a_5+2-2} \cdots z_e^{a_{e-2}-2} z_{e-1}^{a_{e-1}-1}, \quad 4 \leq \delta + 1 < e - 1 \leq e - 1.$$  

In the case $e = 4$ these equations are given by the maximal subdeterminants of the $3 \times 2$-matrix

$$\left( \begin{array}{ccc} z_1 & z_2 & z_3^{a_3-1} \\ z_2 & z_2 z_3^{a_3-1} & z_3^{a_3-1} z_4 \end{array} \right).$$

This is in accordance with (and proved by) Wahl's theorem on equations defining rational singularities [6].

For the computation of $T^1$, the vector space of infinitesimal deformations, we use Pinkham's method [4]. In [1] we reduce the problem to the solution of a (large) system of linear equations and give some examples. A general formula for the dimension of $T^1$ will be proved in [2]:

**Theorem 3.**

$$\dim T^1 = \sum_{e=2}^{e-1} a_e + c,$$
where

\[
    c = \begin{cases} 
    1, & e = 3, \\
    2, & a_3 = 2, \\
    3, & a_3 \geq 3.
\end{cases}
\]

In another forthcoming manuscript we determine the invariants and equations for all remaining quotient surface singularities.

REFERENCES


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