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Differential topology, by Morris W. Hirsch, Springer-Verlag, New York, x + 221 pp.

The study of manifolds is the central theme of topology, and provides the overall motivation. In the early period, which ended around 1940, the methods used were rather intuitive and geometric. One school viewed the manifold primarily as a combinatorial object; algebraic topology developed out of this approach. The other tried to work directly with the differential structure of the manifold; for example, de Rham's theory of differential forms and Morse's theory of calculus of variations in the large. The importance of Morse's approach does not seem to have been fully appreciated at the time, and from the mid-thirties to the mid-fifties was a period of relative neglect for the differential viewpoint. Algebraic topology was progressing by leaps and bounds, during this period, but was little concerned with manifolds as such. It was not until the mid-fifties that it was seen how to use the powerful new techniques which had been discovered to obtain results of a kind which would have excited Poincaré and other pioneers. One such result was undoubtedly Milnor's discovery of the exotic spheres. Another was Thom's theory of cobordism, the first serviceable classification of manifolds to be obtained. These outstanding successes led to a great resurgence of interest in the study of manifolds and the modern phase of differential topology got under way. Hirsch's book is not so much concerned with the modern phase. Most of the results he deals with are over twenty years old and many go back to the thirties and before. Nevertheless the continual process of understanding which has gone on over the years means that clear and concise accounts can now be given.

After a brief introduction the basic notions are introduced in the first chapter (manifolds with boundary are given particular attention throughout). Function-spaces of smooth maps are dealt with next, with originality in the proof of the Baire property for the strong topology and in that of the Morse-Sard theorem on the set of regular values. Sheaf theory in disguise is used to formulate a "globalization theorem" relating the local and global, which is applied in showing that C^r-manifolds ($r > 2$) admit C^∞-structure. Similar ideas are used elsewhere with great advantage, particularly in the proof of the transversality theorem which dominates the third chapter. Vector bundles are dealt with succinctly in the fourth chapter and numerical invariants–degree, intersection number, Euler characteristic–in the fifth. The sixth chapter deals with Morse theory and seventh with the basic notions of cobordism. The last two chapters are concerned with isotopy and the classification of surfaces.

On the whole this is a most readable book. The author has taken considerable trouble with the exposition and has improved on previous accounts in many ways. There are some good diagrams and plenty of exercises. Unfortunately the text contains little in the way of worked examples—the practise as distinct from the theory—and because of this I believe that many students may find these exercises discouragingly difficult. And whatever may be said in the preface, the student is expected to know a little homotopy theory and to be acquainted with the Möbius band, the torus, Klein bottle and so forth. Lapses occur here and there; for instance some key definitions, such as *homotopy*, seem to have been forgotten and others, such as *vector field*, are treated very casually. One would prefer to see more illustrations of the various definitions which come in; for example, one feels that some examples of vector bundles should be discussed before plunging into the general theory. Hirsch's point of view is deliberately restricted by his reliance on certain types of argument–he seems to eschew anything algebraic, such as homology theory–and it would be an improvement if the bibliography included some of the other books in this general area which adopt a different stance. But all in all a most welcome addition to the literature.

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The Hopf bifurcation and its applications, by J. E. Marsden and M. McCracken, Springer-Verlag, New York, 1976, xiii + 408 pp., \$14.80.

A classic example from mechanics should make clear what bifurcation from equilibrium of periodic solutions is. Consider a rigid circular hoop so constrained that it can rotate freely about the vertical axis through its center. Suppose a small ball rests at the bottom of the hoop and is constrained to move on the inside rim. Set the hoop to rotate with frequency ω (about the vertical axis through its center). For small values of ω, the ball stays at the bottom of the hoop and that equilibrium position is stable. However, when ω