proof by Wielandt using a minimax argument gives a ready entry into the arena of these results and makes all these results accessible to reasonable juniors and seniors.

The second axe that I have to grind is that, with all the hullabaloo about applications of algebra to economics, genetics, physics, electrical engineering, and so on, little attempt is made, at the early level, to show how algebra can be applied in *mathematics* itself. Sure, using beginning field theory, or some Galois theory, the question of constructibility by straight edge and compass, or the insolvability of the quintic are presented. Outside of these, very little effort is made to illustrate how algebra, even elementary algebra, can be successfully employed in other parts of mathematics. With a modicum of commutative ring theory nice results in number theory can be obtained. Very few of our students see how configurations in a projective plane translate into algebraic statements about the ring of coordinates of the plane, and how, once this is done, theorems in geometry can be proved by proving theorems about these associated rings. There have been resounding successes in combinatorics using deep results in commutative ring theory. By concentrating on specialized situations of these one might be able to get nice applications of very elementary commutative ring theory to interesting (albeit special) problems in combinatorics. One of my own favorite applications of algebra to number theory is Schur's argument for the Gaussian sums, using the fact that the trace of a matrix A is the sum of its characteristic roots and that the characteristic roots of A^2 are the squares of those of A, together with a hand-dirtying argument (which is healthy for our students to see) at the end.

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Integral geometry and geometric probability, by Luis A. Santaló, Addison-Wesley Publishing Company, Reading, Mass., 1976, xvii + 404 pp., $ 19.50.

Integral geometry was the name coined by Wilhelm Blaschke in 1934 for the classical subject of geometric probability. During that year the author came to Hamburg from Madrid and the reviewer from China, and we sat in Blaschke's course on geometric probability. The main reference was an "Ausarbeitung" of a course by the same name given by G. Herglotz in Göttingen. At the end of 1934 the author found his now famous proofs of the isoperimetric inequality in the plane and Blaschke himself found the fundamental kinematic formula and started a series of papers under the general title of "integral geometry". It was a fruitful and enjoyable year for all concerned.

Integral geometry is exactly 200 years old if we identify its birth with Buffon's solution in 1777 of the needle problem: A needle of length h is placed at random on a plane on which are ruled parallel lines at a distance $D \geqslant h$ apart. Find the probability that it will intersect one of these lines. In fact, the answer is $p = 2h/\pi D$. Experiments were made to determine π on the basis of this result, usually with great accuracy.

Elementary problems on geometric probability are many and are interesting. But until 1928 J. L. Coolidge still held the opinion that the subject is "little more than a plaything". The issue was forced by Bertrand's paradox, which arose from the fact that the measure is not unique for geometric events depending on continuous parameters. Poincaré answered this question neatly by requiring the measures to be invariant under a group of transformations. In almost all practical cases this defines the measure up to a constant factor. Poincaré also realized the importance of the kinematic measure, which nowadays is better understood as the Haar measure of a unimodular Lie group, and exploited it.

The basic reason for integral geometry is the presence of a "duality" in most spaces. Examples are points and lines in the plane, points and geodesics in a Riemannian manifold, points and lattices in R^n, points and horospheres in a symmetric space, etc. The two dual geometric elements are related by a notion of incidence. Given a set in space, the measure of the set of dual elements incident to it gives an important invariant of the set. The classical and simplest example is Crofton's theorem: The measure of the set of lines in the plane meeting an arc β, counted with multiplicities, is equal to twice the length of β. When the same idea is applied to lattices, it gives Siegel's proof of Hlawka's solution of a problem of Minkowski on convex bodies. Radon treated the problem of determining a function on the noneuclidean plane from the integrals of the function over all geodesics. This Radon transform was generalized by Fritz John, and later by Helgason, Gelfand, Graev, Vilenkin, and most recently by Guillemin. It plays an important role in partial differential equations and more general integral operators.

A natural application is to stereology, which deals with a body of methods for the exploration of three-dimensional space when only two-dimensional sections through solid bodies or their projections are available. Clearly stereology is useful in biology, mineralogy, and metallurgy. Recently the ideas and tools of stochastic processes are introduced, bringing the subject back to probability theory.

The strides made in the last four decades are enormous. Integral geometry is no longer a mathematical discipline to be ignored. But the subject has still the happy character that it is not so well known, thus allowing a steady and gentle progress.

As the first volume of an ambitious encyclopedia, the book sets a style. It is a combination of a lucid exposition of the introductory aspects and a complete survey of the area. No topic seems to have been left uncovered. There is also a very complete, but selective, bibliography. The author handled his material with great dexterity and ease. The book should serve as an excellent text for a graduate course on integral geometry. The encyclopedia and the author are to be congratulated for their success.

S. S. CHERN

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Methods of accelerated convergence in nonlinear mechanics, by N. N. Bogoljubov, Ju. A. Mitropoliskii and A. M. Samoilenko, Hindustan Publishing Corporation, Delhi, India, viii + 291 pp., $27.90.

(I) The averaging method–or what is so called today–has its origin in