a bundle's local product structure from being a global product structure. Spivak devotes sixty pages to developing the relationship between characteristic classes and curvature in such a way that the Weil homomorphism is seen to appear naturally. His computations of the de Rham cohomology of $G(n, N)$ are carried out by purely differential geometric methods, using the identification of $G(n, N)$ with a quotient of Lie groups and making no appeal to the advanced machinery of algebraic topology. Several of the book's outstanding virtues are represented in this treatment: it is self-contained; it gives more than cursory attention to classical invariant theory; and it prizes and imparts geometric insight.

After such a detailed discussion of the good things in the *Comprehensive introduction*, perhaps we should also look briefly for flaws. They are of the sort that would be expected in a work of such magnitude written over a relatively short period of time. As Spivak says, "what I have written is a second or third draft of a preliminary version". Indeed, there is evidence that he originally expected to write only two volumes, and that the book simply took over. Thus one can find occasional instances of loose organization, sketchy referencing, and oversight. (The first two volumes have been carefully corrected in the separate Errata given in Volumes 2, 3 and 5; especially out of consideration for graduate students, it might be good to publish the corrections to the later volumes also.) However, these things are minor, and do not detract from the pleasure of the book. Perhaps more importantly, some readers may be disappointed by a certain lack of synthesis, and wish that Spivak had revealed, for the sake of argument at least, what conclusions he has drawn about differential geometry, its history, and its future.

But it would be ungrateful to ask for more than Spivak has already given us. The *Comprehensive introduction* will be widely read and enjoyed, and will surely become a standard reference for graduate courses in differential geometry. Spivak is greatly to be thanked for this spontaneous, exuberant and beautifully geometrical book.

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Modular representations of finite groups, by B. M. Puttaswamaiah and John D. Dixon, Academic Press, New York, San Francisco, London, 1977, vii + 242 pp. \$23.50.

The classical theory of the complex representations of a finite group G can be studied in a number of different but closely related ways. First, one can work with the actual representations, the homomorphisms of G into complex general linear groups. This leads to complex valued functions on G, in particular, the characters, and a very function oriented approach. Another way is the study of the ring theoretic structure of the complex group algebra which leads to idempotents, ideals, left ideals and so on. Finally there is the module approach and the study of homomorphisms, endomorphisms, tensor products. These three different methods exist side by side. For example, there are three approaches to the representations of the symmetric groups: the character method of Frobenius, the idempotents of Young, the Specht modules.

The modular theory of group representations brings in representations over rings other than the complex numbers. Not only do such representations arise naturally in group theoretic problems, but, more importantly, their study leads to new theorems about complex representations. Moreover, new results about the values of characters can be used in deriving the structure of groups. Thus, we can pass from representations in characteristic p to values of characters to structure; this is just another example of where "reduction modulo p" can be exploited.

Modular representation theory uses three rings: a number field K which is a splitting field for G, the localization R of the integers of K at a prime divisor of the rational prime p, the residue class field F of R, which is a finite field of characteristic p. The theory intertwines the representation theory of these three rings. There are also the three different approaches to the representations of G over each of these rings so there are nine distinct situations to be dealt with and interrelated. However, the deeper parts of the subject bring in the subgroups of G, in particular the p-local subgroups of G, that is, the normalizers of the nonidentity p-subgroups of G. Thus, one studies nine distinct situations for G and also for a host of subgroups of G. This makes the subject difficult to learn, harder to master and very hard to teach. But it is a very beautiful, very deep and very useful theory, well worth whatever effort it takes. It is one of the high points of the theory of finite groups as it has developed over the last two hundred years.

There are three types of books that can be written on this theory: a treatise, primarily for the people who know the theory, a monograph on a special topic, an introduction. The authors of this book have written an introduction and have produced the best one available. This book is to be recommended to students desirous of learning the subject.

The basic results of the theory, with the notable exception of the Green correspondence, are all treated in this text. The prerequisites are minimal; some knowledge of groups and rings will suffice. However, it's unlikely that a reader, not already familiar with the classical complex representation theory, could learn the modular theory directly, so that the effective prerequisites are much greater. Similarly, the many nice applications of modular representation theory spread throughout the book can only be appreciated with a prior knowledge of the applications of the classical theory. There are many examples given, either worked out or left to the exercises. However, these examples tend to be very elementary; it would have been very illuminating to have all the details of the theory worked out for the linear fractional groups, p-nilpotent groups or other classes of groups.

Modular representation theory is also called block theory because the idea of a block is so dominant. If p is a prime and G is a finite group then there is a partition of the irreducible characters of G into subsets called the p-blocks of G. This partition is defined in terms of congruences in the values of the characters of G but it is very closely connected with the structure of the group algebra of G over an algebraically closed field of characteristic p. The main aim of the subject is to determine values of characters in a block by using this connection. Each p-block B has associated to it a defect group D which is a subgroup of G of order a power of p and determined up to conjugacy. The remarkable results achieved in the case that D is cyclic constitute the high point of the theory and the motivation for much present research. Even more remarkable is the simple combinatorial idea which ties together all these results and all the characters, Brauer characters, decomposition numbers, Cartan invariants and modules in a very simple way: this is the Brauer tree which is a tree together with a planar embedding.

This cyclic theory is the subject matter of the final chapter of the book. Wisely, in this introductory treatment, the authors restrict themselves to the case where a Sylow p-subgroup P of G is of order p; thus, each p-block has defect group of order one or p. Unfortunately, the Brauer tree is not introduced and the reader will not get a complete understanding of the theory.

However, the results on characters are completely established. Recall that the character table of G is a matrix whose rows are indexed by the irreducible characters of G and whose columns are indexed by the conjugacy classes of G. The entry in the row of the character χ and column of the conjugacy class K is the value $\chi(k)$ of χ on an element k of K. In our case, suppose that the characters of degree not divisible by p are listed first and followed by all the characters of degree divisible by p. Similarly, list first the conjugacy classes of elements of order not divisible by p and then the ones of order divisible by p. In this way we get a partition of the character table of G into four submatrices. The main results are then as follows: the lower right submatrix is zero; the upper right submatrix, apart from some signs, is the same as the upper right submatrix of the character table of the subgroup $N(P)$, the normalizer of the Sylow p-subgroup P. This is a beautiful result, easy to understand and very useful in applications; but a whole theory is needed for its proof!

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Applied nonstandard analysis, by Martin Davis, Wiley, New York, London, Sydney, Toronto, 1977, xii + 181 pp., $16.95.

Introduction to the theory of infinitesimals, by K. D. Stroyan and W. A. J. Luxemburg, Academic Press, New York, San Francisco, London, 1976, xiii + 326 pp., $24.50.

Foundations of infinitesimal calculus, by H. Jerome Keisler, Prindle, Weber & Schmidt, Boston, 1976, ix + 214 pp.

Infinitesimal calculus used to be about infinitesimal numbers. A derivative was the quotient of two infinitesimals; an integral was the sum of infinitely many infinitesimals. Although discredited by the development of $\varepsilon - \delta$ analysis in the nineteenth century, the notion of infinitesimals has never entirely disappeared. Physicists continue to draw little vectors and label them