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Geometry of spheres in normed spaces, by Juan Jorge Schäffer, Lecture Notes in Pure and Appl. Math., vol. 20, Dekker, New York, 1976, vi + 228 pp., $24.50.

Geometric properties of the unit sphere of a Banach space have proved to give much information about the general nature of the space. For example, it has long been known that a Banach space is reflexive if its unit sphere is uniformly convex; this has been strengthened, so that it is now known that X is isomorphic to a space for which no two-dimensional sections of the unit sphere are nearly squares if and only if X is super-reflexive (no nonreflexive space has all its finite-dimensional subspaces "nearly isometric" to subspaces of X). Another spectacular example is the fact that all infinite-dimensional Banach spaces have arbitrarily large finite-dimensional subspaces that are nearly Euclidean, which has been widely useful and revealing. This book contains much new information about certain aspects of the geometry of unit spheres. It might be described as a detailed and comprehensive study of the girth, perimeter, radius, and diameter of unit spheres of Banach spaces. This field is new and interesting, perhaps even weird. It is not yet clear how important it will be for the study of Banach spaces, but it has connections with several concepts of current research interest, e.g., super-reflexivity, the Radon-Nikodým property, infinite trees, and preduals of $L^1(\mu)$-spaces. Although accessible to beginning students, the book seems primarily of value to research mathematicians interested in some of the concepts mentioned in this review. A nonspecialist might be confused by the frequent mixing of important and not-so-important facts.

With the aim of minimizing details and giving a feeling of the type of results involved, it seems best to describe some interesting facts about the girth and perimeter of the unit sphere that can be stated simply, and leave many other facts and related concepts to the serious reader. Given a Banach space X, let $S(X)$ denote the unit sphere of X (the set of all x with $\|x\| = 1$). If X has dimension 2 and C is the circumference of $S(X)$, then $6 \leqslant C \leqslant 8$, $C = 8$ if and only if S is a parallelogram, $C = 6$ if and only if X has a representation in the plane with $S(X)$ a regular hexagon, and $C(X) = C(X^*)$ if X^* is the dual of X. For Banach spaces of dimension at least 2, a metric on $S(X)$ equivalent to that given by the norm is given by letting $\delta(p, q)$ be the infimum of the lengths of curves in $S(X)$ that join p and q. The *girth* and *perimeter* of $S(X)$ are respectively infimum of the lengths of symmetric closed curves in $S(X)$, and the supremum over p of the infimum of the lengths of symmetric closed curves in $S(X)$ through p; it follows that the girth is $2m(X)$ and the perimeter is $2M(X)$, where

$$m(X) = \inf\{\delta(p, -p): p \in S(X)\},$$

$$M(X) = \sup\{\delta(p, -p): p \in S(X)\}.$$

If X is an l^1 sum of $l^1(\Gamma)$ and $L^1(\mu)$ for μ decomposable and atomless, then $m(X) = 2$, and $M(X)$ is 2 or 4 according as $\Gamma = \varnothing$ or $\Gamma \neq \varnothing$. For finite-dimensional spaces, $M(X) = 4$ is equivalent to $S(X)$ being a cylinder or a rhombus. It is conjectured that, if $M(X) = 4$, then either $S(X)$ is a cylinder or a rhombus, or X is not super-reflexive. The extreme situations are understood: $2 \leqslant m(X) \leqslant 4$ for all X, $m(X) = 4$ if and only if X has dimension 2 and $S(X)$ is a parallelogram, and $m(X) > 2$ if and only if X is super-reflexive.

Since super-reflexivity is isomorphically invariant and self-dual, it follows that $m(X) = 2$ is equivalent to $m(Y) = 2$ if X and Y are isomorphic, or if $Y = X^*$. It is conjectured that $m(X) = m(X^*)$ in general, but this is known only for spaces that are Euclidean, are of dimension 2, or are not super-reflexive. For studying the behavior of $m(X)$ under isomorphism, it is useful to introduce $m_*(X)$ and $m^*(X)$, the infimum and supremum of $\{m(Y)$: Y isomorphic to $X\}$. For infinite-dimensional X, $m_*(X) = 2$, and $m^*(X) \leqslant \pi$ with equality holding if X is isomorphic to an Euclidean space. For X_n of dimension n, $m^*(X_n) \to \pi$ as $n \to \infty$, and it is conjectured that $m^*(X_n) = \pi$ if $n > 2$. It is also conjectured that X is isomorphic to an Euclidean space if $m^*(X) = \pi$, and that $m(X) = \pi$ implies X is isomorphic (or possibly isometric) to an Euclidean space.

A *flat space* is a Banach space X for which $m(X) = 2$ by virtue of there being in $S(X)$ a symmetric simple closed curve of length 4; X is *completely flat* if there is such a *girth curve* that spans X, i.e., if X is the closed linear span of the points of the curve. Strangely, interesting flat spaces exist and flatness is related to several concepts of current research interest. The space $l^\infty(\Gamma)$ of bounded functions on Γ is flat if Γ is an infinite set. If T is an infinite Tihonov space and $C(T)$ is the space of bounded continuous real-valued functions on T, then $m[C(T)] = 2$ because of $C(T)$ not being reflexive. Such a space $C(T)$ is flat if and only if $C(T)^*$ is *not* $l^1(\Gamma)$ for any set Γ, so there does *not* exist a p in $S[C(T)]$ with $\delta(p, -p) = 2$ if and only if

$C(T)^*$ is $l^1(\Gamma)$ for some infinite set Γ. For any Banach space X, X^* is flat if X is flat. If X^* is an $L^1(\mu)$-space, then X being flat is equivalent to X^* being flat, which is equivalent to X^* not being $l^1(\Gamma)$ for any Γ. An $L^1(\mu)$ space is completely flat if and only if it is $L^1[0, 1]$. If X is isomorphic to a flat space, then X has an *infinite supported tree* and neither X nor X^* has the Radon-Nikodým property. The use of "completely flat" has strong motivation, because of the following surprising facts: Let s be a spanning girth curve and p be a point of s. Then there is a unique supporting hyperplane H of $S(X)$ at p; p is an interior point of a subset G of $H \cap S(X)$ whose closed affine span is H; for each q in G, $\sup\{\|q - r\|: r \in G\} = 2$; and G is the set of all $(p - q)/\|p - q\|$ for $q \neq p$ and $q \in s$.

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The theory of unitary group representations, by George W. Mackey, Chicago Lectures in Math., Univ. of Chicago Press, Chicago, Ill.,1976, x + 372 pp., $4.95.

It is probably impossible to write a comprehensive book on the theory of unitary representations. The subject, which logically begins in a modest way with complex representations of finite groups, proceeds to general compact groups, and goes on to treat a variety of noncompact groups, is simply too vast. By this time, as a result of the enormous activity in representation theory which began in the late forties and continues unabated, in fact exponentially, to this day, its sometimes alarming and ubiquitous role in a diversity of fields is well established. What is not well established is any agreement about what part or parts of the theory are the most important or how the subject should be organized or presented. At the same time there are disagreements about what open questions should be pursued and the future development of the theory. This naturally causes difficulties for anyone trying to write about representations. The reviewer sometimes envisages the appearance of a new book entitled, *What everyone ought to know about representations* and hordes of representers eagerly rushing out to acquire it, and later returning, disillusioned or angry with what they have found. Authors should also keep in mind that it is probably more difficult for an outsider to learn a substantial segment of representation theory than it is to write about it sensibly. This particular point is admirably put in the forward to Lang's recent book on $SL(2, R)$ in which he states, "It is not easy to get into representation theory, especially for someone interested in number theory, for a number of reasons. First, the general theorems on higher dimensional groups require massive doses of Lie theory. Second, one needs a good background in standard and not so standard analysis on a fairly broad scale. Third, the experts have been writing for each other for so long that the literature is somewhat labyrinthine." This statement is also significant in view of its tacit bias: the general theorems of the subject are either about representations of Lie groups or require some form of Lie theory in their understanding, a point with which the reviewer has considerable sympathy, but surely an indefensible one. The theory of unitary