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INFINITE DIMENSIONAL COMPACTA CONTAINING NO n-DIMENSIONAL ($n \geqslant 1$) SUBSETS

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A compactum (compact metric space) is said to be *strongly infinite dimensional* provided there is a denumerable family $\{(A_k, B_k) | k = 1, 2, \ldots\}$ of pairs of disjoint closed subsets with the property that if, for each k, S_k is a closed subset which separates A_k and B_k, then $\bigcap\{S_k: k = 1, 2, \ldots\} \neq \emptyset$. The Hilbert cube Q is strongly infinite dimensional; let $Q = \prod_{k=1}^{\infty} I_k$ where $I_k = [-1, 1]$, let the projections be denoted by $\pi_k: Q \longrightarrow I_k$, and let $A_k = \pi_k^{-1}(-1)$ and $B_k = \pi_k^{-1}(1)$ (see **[H-W**, p. 49**]**).

In 1965, Henderson **[He-1]**, **[He-2]** constructed a strongly infinite dimensional compactum containing no n-dimensional ($n \geqslant 1$) closed subsets; the following theorem states that there exist strongly infinite dimensional compacta which do not contain any n-dimensional ($n \geqslant 1$) subsets.

THEOREM. *Every strongly infinite dimensional compactum contains a strongly infinite dimensional subcompactum which contains no n-dimensional* ($n \geqslant 1$) *subsets.*

The question of whether or not such examples existed recently had taken on particular importance since Kozlowski **[Ko]** had shown that if no such examples existed, then *CE*-mappings could not raise dimension.

In 1967, Bing **[Bi]** gave a simpler construction of a Henderson-type example. In 1974, Zarelua **[Za]** constructed Henderson-type examples using a different approach. Recently, L. Rubin, R. Schori and the author **[R-S-W]** developed an axiomatic approach for constructing Henderson-type examples; the examples announced in the above theorem are constructed using this axiomatic development.

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