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FINITE RICKART C^*-ALGEBRAS

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1. A C^*-algebra is Rickart [1] if the right annihilator of each element t is generated by a projection $1 - RP(t)$. If in addition, $xx^* = 1$ implies $x^*x = 1$, the ring is called a finite Rickart C^*-algebra. In this note we announce several new results on finite Rickart C^*-algebras. Detailed proofs will appear elsewhere.

Two projections e and f in a Rickart C^*-algebra are $*$-equivalent ($e \overset{*}{\sim} f$) if there exists a w such that $ww^* = e$ and $w^*w = f$. Kaplansky asked whether left projections in a Rickart C^*-algebra were $*$-equivalent to right projections, that is, whether $RP(t) \overset{*}{\sim} LP(t)$ [5]. We have the following partial answer.

THEOREM 1 [2]. *In a finite Rickart C^*-algebra, left projections are equivalent to right projections. In fact $RP(t)$ and $LP(t)$ are unitarily equivalent for each element t in the algebra.*

A consequence of this is the following.

COROLLARY 2 [2]. *A simple homomorphic image of a finite Rickart C^*-algebra is a finite AW^*-factor.*

THEOREM 3 [3]. *If T is a finite Rickart C^*-algebra that is either abelian or an $n \times n$ matrix ring over some ring for $n > 1$, then all matrix rings over T are also finite Rickart C^*-algebras.*

A finite Rickart C^*-algebra is a subdirect product of its simple homomorphic images.

THEOREM 4 [4]. *In a finite Rickart C^*-algebra, the intersection of the maximal (two-sided) ideals is zero.*

Applying Theorem 4 and Corollary 2, we have

COROLLARY 5 [4]. *A finite Rickart C^*-algebra can be embedded in a finite AW^*-algebra.*

2. Outline of the method. In [2] we show that a finite Rickart C^*-algebra T has an $\aleph_0$-continuous (unit) regular quotient ring R to which the involution on T can be lifted. Certain questions concerning finite Rickart C^*-algebras can then be 'lifted' to this quotient ring. This is implemented by a close study

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of the relations between the lattices of projections of R and T. By considering the rank functions on R, and by an examination of the Grothendieck group of an $\aleph_0$-continuous ring as a directed abelian group, we have the following intermediate result.

THEOREM 6 [4]. *In an $\aleph_0$-continuous ring, the intersection of the maximal (two-sided) ideals is zero.*

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