ON A PROBLEM OF ROTA

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Let $S(n, k)$ denote the Stirling numbers of the second kind, and let $K_n$ be such that $S(n, K_n) > S(n, k)$ for all $k$. Rota's problem [3] is to prove or disprove the following:

For all $n$, the largest possible incomparable collection of partitions of an $n$-set contains $S(n, K_n)$ partitions.

An “incomparable collection” of partitions is one in which no partition in the collection is a refinement of some other partition in the collection.

**DEFINITION.** Let $S(n, k)$ denote the collection of all partitions of an $n$-set into $k$ nonempty blocks. If $C \subseteq S(n, k)$, define $\text{Span}(C)$ by

$$\text{Span}(C) = \{\pi \in S(n, k+1): \pi \text{ is a refinement of some } \pi' \in C\}.$$ 

**THEOREM.** For all sufficiently large $n$, there is a collection $C \subseteq S(n, j)$ such that

(i) $j + 1 = K_n$,
(ii) $|\text{Span}(C)| < |C|$, where $|\ |$ denotes cardinality.

Consequently, $(S(n, j + 1) - \text{Span}(C)) \cup C$ is an incomparable collection with more than $S(n, K_n)$ partitions.

**REMARKS.** $C$ consists of all $\pi \in S(n, j)$ having exactly $l$ blocks of size $\leq M$ and exactly $j - l$ blocks of size $> M$ and $\leq 2M$, where $l$ and $M$ are appropriately defined.

The proof of the Theorem requires [2] to estimate $|C|$ and $|\text{Span}(C)|$; and also requires [1] to know the approximate value of $K_n$.

**REFERENCES**

1. E. Rodney Canfield, On the location of the maximum Stirling number(s) of the second kind, 1977 (preprint).

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