bibliography contains papers not referred to in the book itself, but which relate closely to the topics covered and which should provide impetus for further research.

## REFERENCES

[GJ] L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, Princeton, N.J., 1960.

ANNE K. STEINER

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 3, May 1978


*Parallelisms of complete designs*, by Peter J. Cameron, Cambridge Univ. Press, New York, 1976, 144 pp., $ 9.95.

Combinatorics is primarily concerned with two general types of questions concerning arrangements of objects: enumeration, when there are many different arrangements; and the study of structural properties, when the desired arrangements are harder to come by. Of course, there is a large overlap of these two types, and they have some common origins.

There are many relationships between combinatorics and other parts of mathematics. Of special importance for Cameron's book are the relationships with groups, the design of experiments, and coding theory. The relationships with finite groups are fairly obvious and go back to the last century: finiteness implies the use of counting; interesting combinatorial objects will frequently have interesting automorphism groups; and most of the known finite simple groups are intimately related with combinatorial objects on which they act. Cameron's book is primarily concerned with structural properties, just as is much of present-day finite group theory. The structural side of combinatorics also arose in the work on the design of statistical experiments of R. A. Fisher and his successors. More recently, algebraic coding theory has produced new insights into standard combinatorial questions of a structural sort. Many of the best designs and codes have tight structures (and frequently have large automorphism groups), suggesting some sort of classification. This is the point of view espoused in much of this book.

Structure is studied by building up global properties from local information (that is, from configuration "theorems"). Classical examples are the coordinatization theorems for projective spaces of dimension at least 3, and of projective planes in which Desargues' "theorem" is assumed. However, even if a complete classification is unreasonable, it may be possible to associate algebraic objects with suitably restricted combinatorial ones, and then apply standard algebraic techniques.

There are three ways an area of mathematics can be surveyed: by a vast, comprehensive treatise; by a monograph on a small corner of the field; or by a monograph on a cross section. Cameron has chosen the latter method for structural combinatorics. After starting with the seemingly specialized notion of a parallelism of a complete design, he is led into questions concerning finite groups, algebras related to important combinatorial objects, coding theory, and a surprising number of familiar topics in combinatorics and finite geometry. Many of the results presented are taken from recent research of the author; yet, they are discussed and proved lucidly and completely, and natural unsolved problems are continually posed. The effect of all this is that the subject is seen to be very much alive and of value to a wide range of mathematicians.

A parallelism of a complete design is a partition of the $t$-sets of an $n$-set $X$ into "parallel classes" of size $n/t$, such that Playfair's axiom holds. Thus each parallel class itself partitions the $n$-set. The first question that must be answered is: do these exist for any $n$ and any divisor $t$ of $n$? That these always exist is a beautiful theorem of Baranyai. This is proved using the max-flow min-cut theorem for networks, which is itself proved in an appendix. Another natural question has to do with enumeration: asymptotic estimates on the number of parallelisms are given when $t = 2$ using latin squares (discussed in another appendix, including a proof of P. Hall's marriage lemma).

The structural study of parallelisms involves hypothesizing some sort of local uniformity. The "parallelogram property" states that, whenever a $2t$-subset of $X$ contains two parallel $t$-sets, necessarily any two of its complementary $t$-sets are parallel. This is shown to completely characterize a parallelism: except for trivial cases, a parallelism with this property must consist of all lines of an affine space over $GF(2)$ (and $t = 2$), or else is a unique example with $t = 4$ and $n = 24$. The proof of this remarkable result uses Tietäväinen's determination of perfect codes over $GF(2)$, which is given (modulo computation) in an appendix. Another appendix constructs the exceptional example, both from coding and group theoretic points of view. The automorphism group of the exceptional parallelism is the Mathieu group $M_{24}$. The set of unions of pairs of parallel 4-sets forms a Steiner system $S(5, 8, 24)$: each 5-set of the 24-set $X$ is in a unique one of these 8-sets. Thus, very early in the book (pp. 20–22) the relationships between parallelisms and various algebraic and combinatorial questions are observed. In particular, the proof of Tietäväinen's theorem leads to designs, codes and association schemes, topics which reappear throughout the remaining chapters.

From an $S(5, 8, 24)$, an $S(4, 7, 23)$ is obtained by considering all the 8-sets containing a fixed $x \in X$. Since it is this $S(4, 7, 23)$ which gives rise to a perfect code, it is natural to fix a point $x$ of an arbitrary parallelism on a set $X$, and look for a Steiner system. This leads both to refinements of the parallelogram property classification, and to a general discussion of Steiner systems ending with a proof of the uniqueness of $S(4, 7, 23)$ and $S(5, 8, 24)$. Finally, an $S(5, 6, 12)$ is constructed from the perfect code associated with $S(5, 8, 24)$.

A symmetric $(v, k, \lambda)$ design consists of a set of $v$ points, and certain subsets called blocks, such that there are $v$ blocks, $k$ points per block, $k$ blocks per point, $\lambda$ points common to any two blocks, and $\lambda$ blocks containing each pair of points. The case relevant to parallelisms is that of biplanes: symmetric $(v, k, 2)$ designs. However, the general theory is efficiently described, including the Bruck-Ryser-Chowla nonexistence theorem, which places strong restrictions on the parameters $v$, $k$, and $\lambda$ (other than the obvious one $\lambda(v - 1) = k(k - 1)$). Biplanes arise as follows. Fix a block $X$ of a biplane, and assume that for any three points of $X$ there is a point not on $X$ lying on a block with each pair of the three points. (Picture a tetrahedron.) Then each point $y$ not in $X$ produces a partition of $X$ via the set of associated "tetrahedra", and by varying $y$ a parallelism of $X$ results (with $t = 3$). There are only two known examples of such biplanes $B(k)$, with $k = 3$ or 6. A beautiful result of the author is presented: if, in addition, every four points of $X$ lie in a subbiplane $B(6)$, then the biplane can only be $B(6)$. This is proved by a brief argument, in which it is shown that the parallelogram property must hold, and hence that the parallelism is of known type. Additional questions involving parallelisms and biplanes lead to some very special association schemes and metrically regular graphs.

Lurking in the background throughout many of these topics are the group theoretic situations in which many of the combinatorial questions were originally asked. This connection is described in the next to the last chapter. Consider a parallelism of $t$-sets of $X$, and let $G$ be its automorphism group. If $G$ is $(t + 1)$-transitive on $X$ and $|X| > 2t > 2$, then the parallelism is shown to have the parallelogram property (and hence is known) or to be of a unique type with $|X| = 6$, $t = 2$. This result, and its proof, are very typical examples of how large groups can be used in combinatorial situations: use of various stabilizers of one or more points of $X$ leads to local "configuration" properties, which in turn permit purely combinatorial classification theorems to be applied. All the group-theoretic background required is proved (in yet another appendix); moreover, the group-theoretic question which required the preceding theorem is also presented. The discussion of automorphism groups concludes with a brief sketch of the classification of parallelisms for which $G$ acts 2-transitively on the set of parallel classes.

Many open problems are presented throughout the book; indeed, the impression is clearly conveyed that any theorem, however beautiful and complete, easily leads to many problems. The final chapter discusses generalizations of the concept of parallelism. Naturally, large numbers of additional open problems result.

This book is a delight to read. The proofs are slick, but well motivated. It is short and carefully organized. The required background is minimal, being only part of a standard first year graduate algebra course. It would be an excellent way for a graduate student to learn many different techniques, some of which may be difficult, but all of which have clear cut and immediate applications to the main topic being studied.

W. M. KANTOR

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 3, May 1978


*Lectures in semigroups*, by M. Petrich, Akademie-Verlag, Berlin, 1977, viii + 168 pp.

What should a book on semigroups be about? Semigroups, of course, but the subject has long outgrown such a simple answer. In 1961 it was possible for Clifford and Preston [3] to attempt to be comprehensive, provided they stuck very strictly to the algebraic theory and ignored ordered and topological semigroups altogether; but by the time their second volume appeared in 1967