to applications to analysis without first computing even one real probability distribution, be it for a passage time, a hitting probability, an occupation time, or some more involved functional. Secondly, the overall tone of the work is already set in the preface as follows: "The great day of the dedicated solitary researcher is over, if indeed it ever existed. . . . In their stead, concern for the human consequences of scientific and technological achievement must become part of our working lives, . . . Only through organized collective action can this be achieved." This being so, it is easy to imagine why the methods and ideas of a generation of researchers should be presented here in a condensed and transparently clear form, with no suggestion of the effort that must have gone into developing them. Professor Lamperti has indeed done a highly praiseworthy job in providing us with a careful and painless review of stochastic processes. For some readers, however, the work may be a trifle unoriginal. A few more novel calculations, descriptive generalities, or even loose ends, might have alleviated the collective mentality and given the reader more to remember.

FRANK B. KNIGHT

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 4, July 1978

Vector measures, by J. Diestel and J. J. Uhl, Jr., Math. Surveys, no. 15, Amer. Math. Soc., Providence, R.I., 1977, xiii + 322 pp., $35.60.

I am an avid reader of the mystery novels of John Dickson Carr and the Poirot stories of Agatha Christie. I was led to these authors by a keen earlier interest in the works of Edgar Allen Poe and the Sherlock Holmes Stories of Sir Arthur Conan Doyle. Thus, in good faith, I cannot say that this book under review is the most entertaining book I've read; however, I can say that it is the most entertaining mathematics book I've ever read (including a famous measure theory book much enjoyed in my wasted youth). Indeed this serious, but sometimes irreverent, romp through vector measures can be enjoyed even by those misguided souls with a strong dislike for vector valued integration and the geometry of Banach spaces.

I will go so far as to say that the introduction alone is worth the (exorbitant?) price of the book: " . . . shortly after 1936, Dunford was able to recognize the Dunford-Morse theorem and the Clarkson theorem as genuine Radon-Nikodym theorems for the Bochner integral. This was the first Radon-Nikodym theorem for vector measures on abstract measure spaces."

"B. J. Pettis, in 1938, made his contribution to the Orlicz-Pettis theorem for the purpose of proving that weakly countably additive vector measures are norm countably additive."

" . . . Dunford and Pettis, in 1940, built on their earlier work to represent weakly compact operators on L_1 and the general operator from L_1 to a separable dual space by means of a Bochner integral. By means of their integral representation they were able to prove that L_1 has the property now known as the Dunford-Pettis property."

"Then came the war! By the end of the war, the love affair between vector measure theory and Banach space theory had cooled. They began to drift down separate paths. Neither prospered. Much of Banach space theory became lost in the mazes of the theory of locally convex spaces. The work in vector measure theory became little more than formal generalizations of the scalar theory. Representation theory for operators on function spaces became the vogue. But all too often these representation theories gave no new information about the operators they represented. During the fifties and early sixties the theory of vector measures languished in sterility. ..."

How sad! How true!

Although the tone is set by the introduction, the mood of the book is set by the opening sentence of the first chapter: "Grubby set-theoretic manipulations cannot be avoided in measure theory and most of them are found in this chapter." Along the way they think nothing of poking good natured fun at themselves: "Wallowing in a state of ignorance, Uhl rediscovered a theorem [of Rickart]" (p. 39) and the origin of some of their examples: "custom made in 1973 by J. Hagler at Murphy's pub, Champaign, Illinois" (p. 57).

Although this book was written to entertain, its content and purpose are quite serious. The authors have endeavored to present a comprehensive survey of the theory of vector measures and, in the process, present the interplay between properties of Banach spaces and measures taking values in Banach spaces. The connecting device is a technical lemma of Rosenthal.

LEMMA (p. 18). *Let $\mathfrak{F}$ be a field of subsets of the set Ω and let (μ_n) be a uniformly bounded sequence of finitely additive scalar-valued measures defined on $\mathfrak{F}$. Then, if (E_n) is a disjoint sequence of members of $\mathfrak{F}$ and $\varepsilon > 0$, there is a subsequence (E_{n_j}) of (E_n) such that*

$$|\mu_{n_j}|\left[\bigcup_{\substack{K\in\Delta\\ K\neq j}} E_{n_K}\right] < \varepsilon$$

for all finite subsets Δ of the positive integers and all $j = 1, 2, \ldots$.

This measure-theoretic result allows one to prove such results as the Orlicz-Pettis theorem (in any Banach space, weak subseries convergence implies unconditional convergence in norm) (p. 22), the c_0-theorems of Bessaga and Pełczyński (A Banach space contains no copy of c_0 if and only if $\Sigma\, x_n$ is unconditionally convergent whenever $\Sigma\, |f(x_n)| < +\infty$ for each $f \in X^*$. Also, if a conjugate space contains c_0 then it contains l_∞) (p. 22), and the Vatali-Hahn-Saks-Nikodym theorem (A sequence (F_n) of strongly additive X-valued measures on a σ-field Σ is uniformly strongly additive provided $\lim_n F_n(E)$ exists in X-norm for each $E \in \Sigma$) (p. 23). While the Rosenthal lemma links the theories of vector valued measures and the geometry of Banach spaces together, the topic that weaves itself through the entire book and ties these areas together is the Radon-Nikodym property. Let (Ω, Σ, μ) be a finite measure space and X a Banach space. Then X has the Radon-Nikodym property with respect to (Ω, Σ, μ) if for each μ-continuous vector measure G: $\Sigma \to X$ of bounded variation there is a $g \in L_1(\mu, X)$ such that $G(E) = \int_E g\, d\mu$ for all $E \in \Sigma$. The space X has the Radon-Nikodym property, if X has the Radon-Nikodym property with respect to every finite measure space.

Before discussing some of the operator and geometric properties equivalent to the Radon-Nikodym property, it seems worthwhile to reproduce the short history given by Diestel and Uhl: "Some fifty years ago in Latvia, a thirty-year-old mathematician in the School of Railways went to the American Counsulate and claimed he had a job waiting for him at Dartmouth University. In his possession was a post card saying 'the weather at Dartmouth is fine!' It was by this prearranged signal that J. D. Tamarkin was able to find his way to the United States. In the United States, Tamarkin met J. A. Clarkson and suggested that Clarkson look at differentiability properties of vector-valued functions. This was the beginning of the study of the Radon-Nikodym property and led to Clarkson's fundamental paper. Interestingly enough, this paper which is quite geometric in nature has as its avowed object the isolation of *geometric* conditions on a Banach space X that ensure that X-valued functions of bounded variation are differentiable almost everywhere, a condition equivalent to the Radon-Nikodym property. This is how uniformly convex Banach spaces were born."

It is comforting that a Banach space has the Radon-Nikodym property if and only if it has the Radon-Nikodym property relative to $[0, 1]$ with Lebesgue measure (p. 138). The authors amass twenty-three (twenty-nine if X is isomorphic to a conjugate Banach space) statements all equivalent to X having the Radon-Nikodym property.

We look at a few of the more rewarding equivalences.

THEOREM OF LEWIS AND STEGALL (p. 66). *A Banach space X has the Radon-Nikodym property with respect to* (Ω, Σ, μ) *if and only if every bounded linear operator T: $L_1(\mu) \to X$ admits a factorization*

$$\begin{array}{ccc} L_1(\mu) & \xrightarrow{T} & X \\ & {\scriptstyle S}\searrow \quad \nearrow{\scriptstyle L} & \\ & l_1 & \end{array}$$

Perhaps the most geometrically satisfying result is the following:

THEOREM (p. 198). *Let X be a Banach space. Then X^* has the Radon-Nikodym property if and only if X^* has the Krein-Mil'man property.* (*A Banach space has the Krein-Mil'man property if each closed bounded convex subset of X is the norm closed convex hull of its extreme points.*)

Always, a space with the Radon-Nikodym property has the Krein-Mil'man property. It is not known if the converse is true.

There are many nice geometrical results given. These involve technical concepts, e.g. dentability and strongly exposed points, and will not be stated here. However there is a result which should be considered in tandem with the famous characterization of reflexivity of R. C. James. Namely,

THEOREM OF MORRIS AND HUFF (p. 207): *A Banach space X has the Radon-Nikodym property if and only if for each closed bounded subset A of X the collection of $x^* \in X^*$ that attain their maxima on A is norm-dense in X^*.*

Perhaps the most pleasing chapter is Chapter VI on operators on spaces of continuous functions. Although much beautiful work is done here we mention only one result giving an operator characterization of the Radon-Nikodym property.

THEOREM (p. 175). *The space X has the Radon-Nikodym property if and only if for every compact Hausdorff space K, every absolutely summing operator from $C(K)$ to X is nuclear.* (*Recall that T: $X \to Y$ is absolutely summing if it takes unconditionally converging series in X to absolutely convergent series in Y; and T is nuclear if $T = \sum_{n=1}^{\infty} T_n$, each T_n has rank one and $\sum_{n=1}^{\infty} \|T_n\| < +\infty$.*)

Also of interest is the work on Martingales given in Chapter V. Here conditions are given in terms of Martingales for spaces to lack the Radon-Nikodym property.

The format of the book is that of Dunford and Schwartz, i.e. extensive notes and remarks sections. Indeed, these notes and remarks, I feel, are the best parts of the book. The bibliography contains a *thorough* listing of papers pertinent to the subject. A brief rundown of the chapter contents can be found in Notices Amer. Math. Soc. **24** (1977), p. 296.

An annoying aspect of the work is that sometimes the enthusiasm and exuberance for their subject matter leads the authors to some strange sentence constructions. Here are a few examples.

(p. 87) "... Uhl asked whether a separable Banach space X with the Radon-Nikodym property is isomorphic to a subspace of a separable dual space? Stegall showed that if X is a dual space, the answer is yes." One wonders how long it took Stegall to do this!

Fortunately they give a reference to p. 195 where the actual result of Stegall appears. [The authors should be given credit for making Stegall's Joyceian mathematical prose most comprehensible.]

Another interesting statement of the above genera is given on p. 117: "Finally we remark that the closely related problem of characterizing for which Banach spaces X is $L_1(\mu, X)$ weakly sequentially complete is open."

There even appears to be a slip up in their amusing "six lemma" (p. 255):

"Let X and Y be B-spaces and T: $X \to Y$ be a bounded linear operator that admits the factorization

$$\begin{array}{ccc} X & \xrightarrow{\qquad\qquad T \qquad\qquad} & Y \\ S \Big\downarrow & & \Big\uparrow D \\ Z_1 & \underset{I}{\rightarrow} Z_2 \underset{W}{\rightarrow} Z_3 \underset{E}{\rightarrow} Z_4 \underset{L}{\rightarrow} Z_5 \underset{R}{\rightarrow} & Z_6 \end{array}$$

where D, R, L, E, W, I and S are all bounded linear operators and Z_i, $i = 1, 2, 3, 4, 5, 6$, are all Banach spaces of type $L_1(\mu)$, $L_2(\mu)$ or $C(\Omega)$ such that exactly two of the Z_i's are of each type and no type appears consecutively in the above factorization. Then T is nuclear! Of course the converse holds."

I'm not sure what the converse is. If they mean any nuclear operator admits such a factorization then, of course, the result is false! Anyway what is clear about the "six lemma" is that W, J, D, A, V, I and S should object!

Of course it isn't too important but I've always thought that Pitt is responsible for the result that any $T: l_p \to l_q, p > q$, is compact. The authors ascribe this to Paley (without reference). But, enough of this!

The book is highly enjoyable reading for anyone and must reading for anyone interested in vector measures or the geometry of Banach spaces.

The book, like most first editions, has misprints. No one will have difficulty with "language operators" (p. 148) or "1converging" (p. 182) [when read in context] and serious readers will find the subscripts lost or interchanged in some of the displays.

Thus the only serious mistake is the misspelling of the reviewer's name (p. 253).

J. R. RETHERFORD

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 4, July 1978

Jordan pairs, by Ottmar Loos, Lecture Notes in Math., vol. 460, Springer-Verlag, Berlin and New York, 1975, xvi + 218 pp., $9.50.

Jordan pairs are a generalization of Jordan algebras and Jordan triple systems.[1] The archetypal example of a Jordan algebra is the hermitian $n \times n$ matrices $x^* = x$ (for $x^* = \bar{x}^t$ the conjugate transpose) under the product $U(x)y = xyx$, while an example of a Jordan triple system is the rectangular $n \times m$ matrices under $P(x)y = xy^*x$. Such Jordan systems have recently come to play important roles in algebra, geometry, and analysis. In particular, the exceptional Jordan algebra $H_3(K)$ of hermitian 3×3 matrices with entries from the Cayley numbers K has important connections with exceptional geometries, exceptional Lie groups, and exceptional Lie algebras.

Although the structure of finite-dimensional Jordan algebras is well known, the structure of Jordan triple systems is generally known only over algebraically closed fields. The main obstacle to attaining a complete theory for triple systems is the paucity of idempotents: most nonassociative structure theories lean heavily on Peirce decompositions relative to idempotents, and a general triple system may have few "idempotents" x with $P(x)x = x$. For example, the triple system obtained from the real numbers via $P(x)y = -xyx$ has no nonzero idempotents at all. However, a well-behaved triple system does have many pairs of elements (x, y) such that $P(x)y = x$, $P(y)x = y$ (in the above example, for any $x \neq 0$ we may take $y = -x^{-1}$). Such a pair furnishes a pair of simultaneous Peirce-like decompositions of the space, which could provide useful structural information if the two didn't keep getting tangled up in each other.

Even in Jordan algebras, many concepts involve a pair of elements (x, y). Frequently this takes the form of x having a certain property, such as idempotence ($x^2 = x$) or quasi-invertibility (invertibility of $1 - x$), in the y-homotope; this roughly corresponds to the element xy having that particular property, and so serves as a substitute for the associative product xy which doesn't exist within the Jordan structure. (The *y-homotope* of an associative

[1]For a quick background survey of these systems see the article, *Jordan algebras and their applications* in this issue.