In fact, the time seems reasonably near for an historically noteworthy combination of the algebraic theory of near-rings with the fields of nonlinear differential equations, nonlinear functional analysis and numerical analysis. One would be mistaken to dismiss the subject of near-rings as just a haven for out-of-work ring theorists.

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Qualitative analysis of large scale dynamical systems, by Anthony N. Michel and Richard K. Miller, Academic Press, New York, 1977, xv + 289 pp., $22.50.

One of the challenges to system theory posed by present day technological, environmental and societal processes is to overcome the increasing rise and complexity of the relevant mathematical models. The amount of computational effort needed to analyze a dynamic process usually increases much faster than the size of the corresponding systems. Consequently the problems arising in large scale systems become either very difficult or uneconomical to solve even with modern computers. In view of this, it has recently been recognized that, for the purposes of stability analysis, control, optimization and so forth, it may be fruitful to decompose a large scale system into a number of interconnected subsystems, to fully utilize the information of these individual subsystems, and to combine this knowledge with interconnection constraints to obtain a solution of the original problem of the large scale system. Thus taking advantage of the special structural features of a given large scale system, one hopes to devise feasible and efficient piece-by-piece methods for solving such systems which are intractable or impractical to tackle by one shot methods.

It is well known that using a single Lyapunov function and the theory of differential inequalities, a variety of problems in the qualitative theory of differential equations can be studied. However, the usefulness of this approach is limited when applied to problems of higher dimension and complex interconnecting structures. It is also known that, in such situations, employing a vector Lyapunov function instead of a scalar Lyapunov function, is more advantageous since it offers a more flexible mechanism and demands less rigid requirements on each component of the vector Lyapunov functions. Many multivariable systems are composed of relatively simple subsystems or aggregates. By grouping variables of a large economy, for example, into a relatively small number of subeconomies, the economy is decomposed into several interconnected subsystems. Stability of the entire economy may be predicted by testing the lower order aggregated comparison subsystems.

Let E_i, $i = 1, 2, \ldots, N$, be Banach spaces. Consider the system of differential equations

$$x_i' = f_i(t, x_i), \qquad x_i(t_0) = x_{i_0}, \tag{1}$$

where $f_i \in C[R^+ \times E_i, E_i]$. Let $F_i \in C[R^+ \times E, E_i]$ where $E = E_1 \times E_2$

$\times \cdots \times E_n$ and consider the interconnected system

$$x_i' = f_i(t, x_i) + F_i(t, x), \qquad x_i(t_0) = x_{i_0}. \tag{2}$$

Here we have let $x = (x_1, x_2, \ldots, x_n)$. Observing that E is also a Banach space with the induced linear operations and the norm $\|x\| = \|x_1\| + \|x_2\| + \cdots + \|x_n\|$, we can write (2) in the form

$$x' = f(t, x), \qquad x(t_0) = x_0, \tag{3}$$

on the basis of the aggregated stability properties of the subsystems (1) Lyapunov functions $V_i(t, x)$ may be constructed. Then by making use of this constructed vector Lyapunov function $V(t, x)$ and the nature of the interactions between the subsystems, it is possible to predict the stability properties of the large scale system (3) by the method of vector Lyapunov functions. Thus the effectiveness of this method, where multivariability and other structural properties of the interconnected system (3) make the construction of a single Lyapunov function difficult for these systems, is clear.

Significant problems leading to large scale systems arise in diverse disciplines. Typical examples of large scale systems occur for instance, in electrical power systems, aerospace systems, processes in the chemical and petroleum industry, economic systems, ecological systems and societal systems.

The state of the art in the area of qualitative analysis of large scale systems by the method of vector Lyapunov functions has reached a reasonable state. Moreover, problems associated with large scale systems offer new and interesting challenges. As a result a book summarizing the important work done in this field seems desirable. This book does the job.

The book considers many important classes of equations that can be used in modeling a variety of large scale systems. Specifically, large scale systems represented by ordinary differential and difference equations, stochastic differential equations, functional differential equations, Volterra integro-differential equations and certain partial differential equations are discussed in a unified way. Qualitative aspects such as Lyapunov and Lagrange stability, trajectory estimates, input-output properties and questions concerning well posedness are treated. Background material on Lyapunov second method, theory of differential inequalities, theory of M-matrices, semigroup theory and system theory is included. Several specific examples are given to illustrate the results. However, the involvement of the book with large scale systems does not extend beyond the title. The only two specific examples discussed that represent large scale systems do not seem to have meaningful applications.

The readers interested in the method of vector Lyapunov functions should consult the original work of Matrosov [1] and other important contributions [2]–[7] (which are not referred to in the book) for more insight into the historical development and applicability of the method. In addition, an excellent survey paper by Siljak [8] and several other contributions of merit [9], [10], [12]–[18] on the application of the method of vector Lyapunov functions to realistic problems in large scale systems should deserve attention.

This book combined with the book on the same subject by Siljak [**11**] which is also just published and which presents several specific applications of large scale systems to real problems, should form a welcome addition to the subject and help stimulate further work in this new and exciting field.

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Linear representations of finite groups, by Jean-Pierre Serre, Graduate Texts in Mathematics, vol. 42, Springer-Verlag, New York, Heidelberg, Berlin, 1977, x + 170 pp., $12.80.

As one so often discovers in tracing the history of a beautiful mathematical idea, we have Gauss to thank for pointing the way towards the representation theory of finite groups. In his discussion of class groups of even binary quadratic forms of given discriminant over the integers, he attempted to