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First order categorical logic, by Michael Makkai and Gonzalo E. Reyes, Lecture Notes in Math., vol. 611, Springer-Verlag, Berlin, Heidelberg, New York, 1977, viii + 301 pp. $14.30.

The authors obtained new proofs for theorems of Barr and Deligne concerning topoi, and also obtained some basic new results in this area. Their proofs are applications of standard results and techniques of logic. Since the relationships between logic and category theory at this relatively deep level are not widely known, they wisely decided to give a general and self-contained explanation of these relationships in addition to their proofs and results. The resulting work, despite its unfinished character typical of the lecture notes series, should hence be, at least in part, of considerably wider interest than a research paper on these results would have been.

Before describing briefly the contents of the book, it is desirable to make some remarks on the role of categorical logic in mathematics. Categorical logic may be described as one of the algebraic ways of looking at logic. Algebraic logic arose, in part at least, from trying to conceive of logical notions and theorems in terms of universal algebraic concepts. Thus polyadic algebras (see Halmos [**Hal**]) and cylindric algebras (see Henkin, Monk, Tarski [**HMT**]) are algebraic versions of logic, and are algebraic structures (certain Boolean algebras with operators) which can be, and have been, studied in much the same way that one studies groups, rings, lattices, etc. At the same time that algebraic logic in this sense has been developing, category theory also developed, and in particular many facets of universal algebra were generalized (see Mac Lane [**Mc L**]). The interplay between category theory and logic may be considered to be another algebraization of logic. The present book gives one of the first systematic treatments of this kind of algebraization.

The main motivation for the study of algebraic logic so far has been just the intrinsic beauty of treating on a general, algebraic level matters which are very concrete and suspiciously philosophical. The present work, giving applications to theorems and notions relevant to algebraic geometry, provides additional motivation for the study of the subject.

The book should be of interest to category theorists and to logicians, although the reviewer feels that it will be more accessible to the former. Indeed, the background required to read it seems to be a basic knowledge of category theory, say half of [Mc L], but no logic. To understand it well, though, one should be familiar with topoi (as in [AGV]) and should know some model theory, say half of [CK].

The basic notion of categorical logic can be explained briefly to those knowing some category theory and logic. Let L be a first-order language and let C be a category with finite limits. A *C-structure of type L* is a pair $\langle A, F \rangle$ consisting of an object A of C and a function F. F assigns to each n-ary relation symbol R of L a subobject of $A \times \cdots \times A$ (n times), and to each n-ary operation symbol 0 of L an arrow $A \times \cdots \times A \to A$, with n A's. It is then routine to define satisfaction of formulas with respect to such structures, mimicing the usual case $C = \text{Set}$. To make sure that the semantics are well behaved one has to put additional restrictions on C, leading to the notion of a logical category, which comes close to the notion of a topos in [AGV]. One can then formulate completeness theorems, etc., in categorical terms.

A brief survey of the book's contents are now in order. The Leitfaden on p. 10 is very useful, and in this survey we follow an order the reviewer finds more natural than the linear one. In Chapter 4 the basic semantic notions of many-sorted $L_{\infty\omega}$ are introduced. Gentzen sequents are interpreted in Heyting-valued models with empty universes admitted. In Chapter 5 Mansfield's Boolean-valued completeness theorem is given. In Chapter 2 the basic notion of a categorical interpretation of a language is given (see above for a simplified version). Conversely, given a category C, one can form an associated language L_C by letting the sorts of L_C be the objects of C and the arrows $f\colon c \to d$ of C be unary operation symbols of sort $\langle c, d \rangle$. Many categorical notions in C can be expressed by satisfaction of suitable sequents in L_C; for example, $fx = fy \Rightarrow x = y$ expresses that f is monic. In Chapter 3 additional restrictions are put on C, leading to a *soundness* theorem (roughly, if σ formally follows from T and T holds in C, then σ holds in C), and a completeness theorem for logical categories (due to Deligne-Joyal), which says that for any logical category C there is a set I and a faithful logical functor: $C \to \text{Set}^I$. Chapter 7 contains algebraic forms of some logical theorems, e.g., Beth's theorem and the Łoś-Tarski theorem, and characterizes the notion of a pretopos (the latter is one of the main new results in the book). In Chapter 8, it is essentially shown how to go back and forth from logic to category theory; this is the basic chapter from the algebraic logic point of view. Chapter 1 has a self-contained exposition of topoi, following [AGV]. Chapter 6 contains the new proofs of Barr's and Deligne's theorems. Finally, Chapter 9 summarizes some of the preceding material in different terms.

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