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Georg Cantor, his mathematics and philosophy of the infinite, by Joseph Warren Dauben, Harvard Univ. Press, Cambridge, Mass., 1979, ix + 404 pp., $28.50.

A century and a half ago in 1831 Gauss, in a letter to Schumacher, wrote: "I protest against an infinite quantity as an actual entity; this is never allowed in mathematics. The infinite is only a manner of speaking."

Forty-one years later Georg Cantor, a young mathematician at Halle, was studying the uniqueness problem for trigonometric series. In 1870 he had proven that if a real function f was represented by a trigonometric series which converged for all x, then the series was necessarily unique; in fact, uniqueness was guaranteed even if the set of exceptional points, where convergence failed, was discrete. By the following year he had extended his uniqueness theorem to the case where the set of exceptional points possessed finitely many limit points.

It was in investigating the case where the set of exceptional points possessed infinitely many limit points that Cantor made the seminal discovery of his career. Given a point set P he had defined its derived set P' to be the set of limit points of P and had gone on to iterate the process, obtaining in this way the sequence $P, P', P^{(2)}, \ldots, P^{(n)}, \ldots$. He was successful in modifying his earlier proofs and was able to establish uniqueness of representation anytime the set of exceptional points P had the property that $P^{(n)}$ was finite for some n. But he also discovered point sets P with the property that $P^{(n)}$ was infinite for all n;[1] to understand these sets he was forced to introduce derived sets of infinite order $P^{(\infty)}$, $P^{(\infty+1)}$, and so on. This discovery marked the beginning of the occurrence of the actual infinite in mathematics; as Cantor wrote in 1872: "The number concept, in so far as it is developed here, carries within it the germ of a necessary and absolute extension."

This discovery also marked the beginning of a new perception of the role of the mathematician. For when Cantor turned his telescope to the infinite, one of the first things he discovered was that there were no words available to describe what he saw. Thus in order to understand and give meaning to what he saw he found it necessary to create the very lexicon–the pure symbolic forms–through which this understanding was to come.

Joseph Dauben's intellectual biography of Cantor is a very detailed exposition of the evolution of these symbolic forms. It is remarkable that all of the following concepts had their genesis in Cantor's works: The real line (defined in terms of Cauchy sequences); limit points; neighborhoods; derived sets; connected sets; closed sets; everywhere dense sets; nowhere dense sets; perfect sets; countable sets; the value of one-one correspondences; cardinal numbers; order types and well-ordered sets; ordinal numbers; the powers $\aleph_0$, $\aleph_1$ and $\mathbf{C}$; the countable ordinals, including the ϵ-numbers; transfinite arithmetic, both ordinal and cardinal; power sets; and the abstract concept of a set. And he not only created the words through which these concepts could be studied, he proved many of the now standard theorems about them. Only a proof of the continuum hypothesis eluded him.

As is well known, Cantor's introduction of these new symbolic forms met with opposition and even outright hostility. Kronecker was an early and vigorous opponent of his work and Dauben documents Kronecker's attempts during the 1870s to prevent, or at least delay, the publication of many of Cantor's papers; in particular his 1878 paper *Ein Beitrag zur Mannigfaltigkeitslehre* in which appeared Cantor's proof that the plane and the line were equinumerous. On the other hand, Dauben claims that Cantor's choice of title for his 1874 paper, *Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen* was made so as to minimize conflict with Kronecker; for

[1]The following examples are due to Mittag-Leffler: (a) Let P be the set of points of the form $1/2^{n+1} + 1/2^{n+1+m_0} + 1/2^{n+1+m_0+m_1} + \cdots + 1/2^{n+1+m_0+m_1+\cdots+m_n}$ where $n, m_0, \ldots, m_n$ range over the natural numbers. Then each $P^{(n)}$ contains $1/2^{n+1}, 1/2^{n+2}, \ldots$ and $P^{(\omega)} = \bigcap P^{(n)} = \{0\}$. (b) Let P be the set of points of the form $1/2^m + 1/2^{m+p} + 1/2^{m+p+q_0} + \cdots + 1/2^{m+p+q_0+\cdots+q_p}$ where $m, p, q_0, \ldots, q_p$ again range over the natural numbers. In this example even $P^{(\omega)}$ is infinite while $P^{(\omega+1)} = \{0\}$.

the major theorem of the paper was not the proof of the denumerability of the set of algebraic numbers, but the nondenumerability of the reals.

Earlier historians, E. T. Bell in particular, have claimed that antisemitism was at the root of much of the opposition to Cantor's work. But as Dauben clearly establishes, Georg Cantor was not of Jewish ancestry; he was baptized a Lutheran and remained a devout Christian throughout his life. We obtain a deeper understanding of the nature of modern mathematics if we look to the mathematician *qua* mathematician for the source of the opposition.

It is consistent with the known facts that Kronecker's unwavering opposition to Cantor's work was the result of a total and fundamental difference of opinion as to the nature of mathematics. The extent of this difference can be seen in two aphorisms: Kronecker's "God made the integers; all else is the work of man" and Cantor's "The essence of mathematics is its freedom". For Kronecker the objects of mathematical investigation were the integers; these were fixed and unchanging. The mathematician's role was limited to the investigation of constructions built upon these eternal god-created forms. Creativity of new forms was not part of the province of the mathematician.

Cantor saw things differently. He knew that he could understand only if he had the freedom to create the forms and concepts which would encapsulate what he sought to understand. Dauben recognizes this, writing that the most important feature of his mathematical ability was "the capacity for creating new forms and concepts when existing approaches failed".

If we are to fully understand Cantor's influence on the nature of mathematical activity it is necessary to see Kronecker as belonging to the mathematical mainstream. It may be true that in his insistence that only the integers possessed an independent existence, he cast his net too narrowly, but the prevailing mathematical opinion then, as it had been since before Plato, was that the essence of mathematical activity is investigative, not creative. Philosophers still hold to this view, being far more concerned with epistemological matters than with ontological ones. However, after initial opposition, mathematicians were quick to appreciate the freedom that Cantor's conception of mathematics offered; as Hilbert wrote in 1925: "No one shall expel us from the paradise which Cantor created for us".

Just as Prometheus stole fire from the gods and instructed the human race in its use, so Cantor showed us that, like Kronecker's God, we too are free to create symbolic forms. The integers may be theogenic; since Cantor the rest of mathematics has become anthropogenic.

STANLEY H. STAHL

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Integral representations of functions and imbedding theorems, by Oleg V. Besov, Valentin P. Il'in, and Sergei M. Nikol'skiĭ, with an introduction by Mitchell H. Taibleson, V. H. Winston & Sons, Washington, D. C., vol. I, 1978, viii + 245 pp., vol. II, 1979, viii + 311 pp., $19.95 per volume.

This book (hereinafter referred to as *Integral representations*) is closely related to, but (both in technique and content) independent of Nikol'skiĭ's [5]