already have a sufficiency of the basic ideas like the book; they can see that while any book of this length may appear "formidable" globally, this particular one is not formidable locally. Therefore, this book is a success with them.

To conclude, I think George Whitehead proves at least one thing we already knew: in calibre as a mathematician, and for qualities of taste, style and judgement, he rates a lot higher than many who have written books on algebraic topology. I welcome this book most warmly, and I look forward to the second volume.

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Percy Alexander MacMahon: *Collected papers*, Volume I, *Combinatorics*, edited by George E. Andrews, Mathematicians of Our Time Series, The MIT Press, Cambridge, Massachusetts, 1978, xxx + 1438 pp., $75.00.

This is possibly a unique presentation of collected papers. First, papers are collected in chapters according to their consonance with the sections of MacMahon's treatise, Combinatory Analysis, whose order they follow. Each chapter begins with an (editorial) Introduction and Commentary, followed by a presentation of related current work, References (sizable bibliographies) and Summaries of the Papers (sometimes including alternative proofs of theorems). The editor's diligence and stamina are most impressive.

The papers, like the treatise, are motivated by MacMahon's ambitious objective (expressed in the preface to the second volume of the treatise) of "presentation of processes of great generality, and of new ideas, which have not up to the present time found a place in any book in any language".

Shorn of its context, this statement may appear more brashly confident than its author intended. It is preceded by a lengthy and glowing appreciation of Eugen Netto's *Kombinatorik*, possibly only to show that he has chosen a different path with due deliberation.

Nevertheless, the pursuit of originality and generality has its perils. For one thing, the current spate of combinatorial mappings has produced the feeling that multiplicity abounds. Perhaps the simplest example is the continuing appearances of the Catalan numbers ($(2n)!/n!(n+1)!$, $n = 0, 1, \ldots$), whose number sequence (No. 577 in Neil Sloane's *Handbook of integer sequences*, Academic Press, 1973) is $1, 1, 2, 5, 14, 42, \ldots$. Incidentally, these numbers are named after E. Catalan because of a citation in Netto's *Kombinatorik*, in relation to perhaps the simplest bracketing problem, proposed in 1838. An earlier appearance, which I first learned from Henry Gould, is due to the Euler trio, Euler-Fuss-Segner, dated 1761. There are now at least forty mappings, hence, forty diverse settings for this sequence; worse still, no end seems in sight. In this light, the Catalan (or Euler-Fuss-Segner) originality may be regarded as temporary blindness.

As for generality, MacMahon (paper 52, Chapter 7), gives a general solution of the Latin square problem–requiring the determination of coefficients in the expansion of powers of a multivariable sum. Since current knowledge of the actual numbers extends to only 9×9 squares, the use of MacMahon's result by mere mortals seems doomed.

On the other hand, MacMahon's work on Latin squares seems to have inspired J. H. Redfield (in a long neglected paper reprinted in Chapter 7) to develop interrelations of Group Theory and Combinatorics, implicit in George Polya's (later) masterly paper on groups, graphs, and chemical structures.

In most of MacMahon's papers, an enumerative aspect is present, at least at the start. In many of the current works cited by the editor, the enumerative side seems strikingly reduced, e.g. in Gian-Carlo Rota's series *On the foundations of combinatorial theory*. In L. Mirsky's brilliant review of *Combinatorics in the theory of graphs* (this Bulletin, March 1979), enumeration is mentioned only to be dismissed. Perhaps it is true that enumeration is obsolescent; true or not, I focus my remaining remarks on ignored enumeration aspects.

Papers 40 and 42, both in Chapter 5, are mainly concerned with a problem whose engineering description is: the enumeration of unlabeled series–parallel networks. In Paper 42, which appeared in the Electrician in 1892, the terms series and parallel appear; in Paper 40, appearing in Proc. London Math. Soc., series becomes chain, parallel becomes yoke, following a suggestion from Professor Cayley, and probably a necessary bow to the referee's sensibility. By a simple and elegant argument, MacMahon derives a generating function identity for the enumerator of these yoke-chains identical with that for a class of trees in Cayley's paper *On the theory of the analytical forms called trees* (*Collected papers*, No. 203); these trees are secondary to the main result which is: the enumeration of rooted trees by number of points with all points alike (i.e. unlabeled). They are identified by the cryptic description: "trees with a given number of free branches, bifurcations at least". MacMahon's diagrams, arranged by number of endpoints, reveal that there are no branches at the root, or any other branch point, represented by two or more lines in series (or tandem), like  . . . . For brevity, I have been calling such trees series-reduced. Moreover they are the trees appearing in Louis Comtet's mapping of the bracketings in E. Schröder's fourth problem reported in C. R. Acad. Sci. Paris Ser. A-B, **271** (1970), 913–916. Curiously Comtet's brief tag, arboresences bifurcante, echoes Cayley. The mapping of chain-yoke arrangements to Schröder's bracketings is sketched in my 1976 Acta. Math. paper.

Paper 40 is in two parts; the second part is concerned with another Cayley tree enumeration (*Collected papers*, No. 297), the trees in question being those without the series-reduced prescription and again enumerated by number of end points. MacMahon gives a mapping of compositions of multipartite numbers with unit parts, zero parts in the composition not excluded.

Cayley's main result is the exponential generating function

$$(2 - e^{x})^{-1} = \sum_{0} \phi(m + 1)x^{m}/m!$$

with $\phi(m)$ the number of trees having m endpoints. This entails, as Cayley found,

$$\phi(m+1) = \sum_0 k!\,S(m,k)$$

with $S(m, k)$ a Stirling number of the second kind. A second result follows from the following relation for the Euler generating function, from which issue the Eulerian numbers, namely

$$H(y,x) = (1-y)(e^x - y)^{-1} = \sum_0 A_n(y)(y-1)^{-n}x^n/n!$$

with $A_0(y) = 1$, $A_n(y) = \Sigma_0^n A_{nk}y^{k-1}$, A_{nk} an Eulerian number. Hence

$$H(2,x) = (2-e^x)^{-1} = \sum_0 A_n(2)x^n/n!$$

and

$$\phi(m+1) = A_m(2).$$

Finally it may be noted that Cayley's generating function implies the recurrence

$$\phi(n+1) = \sum_1^n \binom{n}{k}\phi(n+1-k).$$

I hope that these remarks are sufficient to show that enumeration is not simply a home for mad calculators, even those like me who are allergic to the use of computers. The success of Sloane's *Handbook of integer sequences*, in my view, shows that number sequences have a recognition factor far superior to detailed description of their settings, or even of the mathematical operations by which they are obtained. I have a private dictum: the objective of enumeration is numbers, not formulas. In other words, the latter must be reduced to computation. More than that, the reliability of the reduction must be verifiable, a condition that encourages the use of relevant congruences, allowing a mere enumerator to approach the number theory empyrean.

Not to leave the hard work of the editor in total disregard, I must note that his introductory remarks in Chapter 9 contain fascinating historical material on the Hardy-Ramanujan asymptotic formula for the number of partitions (of numbers), famous for its unexpected property of being exact for sufficiently large n. Indeed, with its help, D. H. Lehmer made the first calculation of the number partitions of 721. MacMahon's part in this was the production of a table with a range of 200 and Hardy is quoted as supplying the characterization: "a practised and enthusiastic computer".

Indeed, partitions seem to have had a special place in MacMahon's heart, judging by the variety of special cases in §VII of the treatise (corresponding to Chapter 9 of the papers), all examined with both elegance and enthusiasm. One interesting offshoot is the appearance of a notation due to Cayley, now familiar as a q-binomial coefficient.

The volume probably has other surprises for the patient reader. One constant source of pleasure (and envy) is the ample space for comfortable exposition available in this not so distant past. If those ignoring the past are due to repeat it, the prudent reader will surely find some way, short of page by page digestion, of taking something of value from this massive tone.

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