The Burnside problem for matrix groups was solved by Schur (1911) who proved that a finitely generated periodic matrix group over C is finite. This was generalized to arbitrary fields by Kaplansky (1965) and to semigroups by McNaughton and Zalcstein (1975). The author presents the latter result using methods of G. Jacob, and in the opposite direction gives examples of infinite sequences without repeats (Morse-Hedlund sequence, though his actual examples follow J. Leech and F. Dejean).

The final chapter is devoted to Foata's theory of "rearrangement monoids", which leads to a quick and transparent proof of MacMahon's Master Theorem, and there are some illustrations from matching problems. The topic is closely related to factorizations in free monoids and bases in free Lie algebras, and these connexions are briefly sketched.

The writing is terse but clear; the many worked illustrations and exercises are particularly useful. While the first four chapters form an excellent introduction to semigroup theory, the account of language theory that follows it is inevitably somewhat biased, being seen from the viewpoint of semigroups. This would make it rather hard as a first introduction, but for anyone with even a nodding acquaintance of language theory it provides an interesting attempt to present an integrated account of these topics. It is only partly successful because the author has perhaps spent a disproportionate amount of space on rather recondite properties of languages because they happen to involve semigroups. However, this is more than made good by the many simplifications, and new applications of semigroup theory. There are one or two slips, of logic (isomorphism of composition series is so defined as to be nonreflexive), spelling (p. 91 preversed, p. 333 envelopping), a curious confusion of left and right on p. 4 and a definition of generating set which quite outdoes Bourbaki (see also the rather bizarre definition of mathematical problem, p. 127). But these are trifles which do not in any way detract from a highly readable book, which will surely help to kill the myth that there are no exciting results in semigroup theory.

REFERENCES

1. A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*. I, II, Math. Surveys, No. 7, Amer. Math. Soc., Providence, R. I., 1961, 1967.

2. S. Eilenberg, *Automata, languages and machines*. A, B, Academic Press, New York and London, 1974, 1976.

3. J. M. Howie, *An introduction to semigroup theory*, LMS Monographs, No. 7, Academic Press, London and New York, 1976.

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The representation theory of the symmetric groups, by G. D. James, Lecture Notes in Mathematics, Volume 682, Springer-Verlag, Berlin and New York, 1978, vi + 156 pp.

Since every finite group G is a subgroup of some symmetric group $\mathfrak{S}_n$, consisting of all the $n!$ permutations on n objects, the representations of $\mathfrak{S}_n$

play an important role in finite group theory. Fundamental work in the representation theory of $\mathfrak{S}_n$ was pioneered by G. Frobenius and Alfred Young. An important book on the subject was written in 1961 by Young's student Gilbert de B. Robinson, who also compiled and edited a volume of Young's collected works, published in 1978. Gordon James, of Cambridge, England, author of the book being reviewed, was an invited speaker at a Waterloo, Ontario, "Young Day" conference celebrating Young's work, and spoke on the decomposition matrices for modular representations, described near the end of his book.

A finite multiplicative group G is usually represented by mapping its elements onto permutations, or linear transformations of a vector space M over a field F, or by matrices describing such permutations or transformations, in such a way that products are mapped into corresponding products. The vector space on which G acts is called an FG-module. It is irreducible if no proper subspace is invariant under all elements of G, and indecomposable if no pair of complementary subspaces are invariant. The trace of a matrix A is the sum of its diagonal entries a_{ii}, and is equal to the trace of any similar matrix $B^{-1}AB$. Hence, in a matrix representation the trace is independent of the choice of basis in M, and is the same for conjugate elements g_i and $g_j^{-1}g_ig_j$ of G. For each representation ρ the set of traces χ_κ^ρ of representative matrices, one for each class C_κ of G, form a class function, written as a vector, and called the character χ^ρ of ρ. The value χ_1^ρ for the identity class is the degree of ρ, and the dimension of the module M which affords ρ. If G is finite and F is the complex number field, all values χ_κ^ρ are algebraic integers that are sums of χ_1^ρ roots of unity. Then the number of equivalence classes of irreducible representations or of irreducible characters χ^ρ is equal to the number of conjugacy classes C_κ of G, and the numbers χ_κ^ρ form a square array called the character table of G.

Irreducible representations and conjugacy classes of the symmetric group $\mathfrak{S}_n$ are each labeled by a partition λ of n, consisting of a set of nonnegative integers λ_i with sum n. If permutations of $\mathfrak{S}_n$ are represented as products of disjoint cycles, the cycle lengths form a partition of n which describes the class. For example, the permutation (1)(2)(34)(5678)(9) belongs to the class $1^3 2 \cdot 4$ of $\mathfrak{S}_9$. Each irreducible representation of $\mathfrak{S}_n$ is labeled by an ordered partition λ, with $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_r > 0$, displayed by a Young diagram $[\lambda]$, that is a left adjusted array of n dots or nodes in r rows, with λ_i equally spaced nodes in row i. If λ_j' nodes appear in column j, the partitions λ and λ' are called conjugate partitions, and $[\lambda]$, $[\lambda']$ are associated diagrams. Each assignment of the integers 1 to n to distinct nodes of $[\lambda]$ determines a λ-tableau t. The tableau is standard if the integers increase from right to left in rows, and from top to bottom in columns. Surprising, perhaps, is the fact that the degree of the representation λ is precisely the number of standard tableaux for λ. The row stabilizer R_t and column stabilizer C_t of a tableau t are respectively the subgroups of $\mathfrak{S}_n$ that leave the integers in the same row, or in the same column, of t. The sum ρ_t of the permutations in R_t, and the sum of the even permutations minus the odd permutations in C_t, called κ_t, are elements of the group ring of G that played key roles in the studies by Frobenius, Schur, Young, and others, of the representations of $\mathfrak{S}_n$.

After discussing modules, submodules, and the classes of $\mathfrak{S}_n$, the author considers $F\mathfrak{S}_n$-modules M^λ for each partition λ of n. A tabloid $\{t\}$ is "a tableau with unordered row entries", and the polytabloid associated with t is defined by $e_t = \{t\}\kappa_t$. The Specht module S^λ is the submodule of M^λ spanned by polytabloids, and is irreducible if char $F = 0$. The standard polytabloids e_t furnish a basis for S^λ. Young's Rule states that the multiplicity of S_Q^λ as a composition factor of M_Q^μ equals the number of semistandard λ-tableaux of type μ. These are generalized λ-tableaux in which the number i appears with multiplicity μ_i, and the numbers are nondecreasing in rows, but increasing downward in columns, and such that $\lambda \trianglerighteq \mu$ under a partial ordering of partitions. Calculation of the components of a product $[\lambda][\mu]$ is facilitated by an algorithm called the Littlewood-Richardson Rule, which also enables one to calculate the composition factors of any ordinary representation of a Young subgroup, induced up to $\mathfrak{S}_n$.

The degree of the irreducible representation $[\lambda]$, which is the dimension of the Specht module S^λ, is given by the simple "hook formula" $n!/H^\lambda$, where $H^\lambda = \Pi_{i,j}h_{ij}^\lambda$, and h_{ij}^λ is the length $1 + (\lambda_i - j) + (\lambda_j' - i)$ of the (i,j)-hook in the diagram $[\lambda]$. Also $e_t\rho_t\kappa_t = H^\lambda e_t$, if $F = Q$. Calculation of single entries in the character table can be done efficiently by extracting from the diagram by the Murnaghan-Nakayama Rule certain hooks that correspond to cycles in the permutation. The character χ^λ vanishes on all permutations whose order is divisible by a prime p if no hook number h_{ij}^λ is divisible by p.

Considerable attention is devoted in the book to the modular representations of S_n over the p element field F_p. Defining the g.c.d. of $\langle e_t, e_{t^*}\rangle$, for polytabloids e_t and e_{t^*} in S_Q^μ, to be g^μ, and μ' to be the conjugate of a p-regular partition μ, it is proved that S^μ is reducible if and only if p divides $H^\mu/g^{\mu'}$. A related criterion for reducibility examines the p-power diagram $[\mu]^p$ obtained by replacing each hook number h_{ij}^μ by the exponent $\nu_p(h_{ij}^\mu)$ of the highest power of p that divides h_{ij}^μ. R. W. Carter has conjectured, and James partially proves, that no column of $[\mu]^p$ contains two different numbers if and only if μ is p-regular and S^μ is irreducible over F_p. Decomposition matrices for modular representations are discussed in detail, and are tabulated in an appendix for $p = 2$ and 3 and $n \leqslant 13$.

The author feels that the representation theory of S_n should be presented without reference to the representing matrices, but he does describe some matrices of Young's orthogonal form near the end of the book. Following this is a discussion of the close relationship between the irreducible representations of S_n and those of the general linear group of dimension d, each corresponding to a partition λ, and acting on a vector space W^λ whose dimension is the number of semi-standard λ-tableaux, with integer entries $\leqslant d$, computable by the formula $\dim W^\lambda = (\Pi(d + j - i))/H^\lambda$, where the product is over all nodes (i, j) in $[\lambda]$.

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