natural invariant for distiguishing algebras is present. A. Connes showed that all possible subgroups occur in practice by forming a new crossed product algebra out of an algebra and a given automorphism group of the algebra. M. Takesaki then showed that every type III algebra is the crossed product of a type II_∞ algebra with a one-parameter automorphism group.

Now the problem of classifying von Neumann algebras reduces to that of classifying type II_1 algebras. Some important strides have already been made by A. Connes again by studying automorphism groups.

The main feature and value of the book *Lectures on von Neumann algebras* is the discussion in Chapters 9 and 10 of modular automorphisms, Hilbert algebras, modular Hilbert algebras and self-polar forms. These are developed for normal semi-finite weights in a manner that parallels the development for quasi-unitary algebras from traces. (The prototype for this is M_n acting on the Hilbert space M_n.) Included is a complete discussion of the theory of closed operators on Hilbert space that is necessary for the discussion of the modular operator. The discussion lacks a little of the clarity of earlier chapters in that much information is carried in general discussion paragraphs rather than in definitions or theorems. Yet the value of having the information all in one place with one set of notation is certainly very great.

The discussion concerning different types of von Neumann algebras is limited to the definition of the different types of algebras and the definition of their basic properties as found in other texts. Connes' classfication of factors is discussed briefly in the appendix of Chapter 10. In fact, two sections (a section E and C) are appended to each chapter: The first being exercises or additional theorems that the reader has a chance of doing for himself and the second being additional related topics that are briefly discussed.

There is no discussion of abelian von Neumann algebras or direct integrals.

Finally, the bibliography has over 120 pages.

Reports of new books on von Neumann algebras and C^*-algebras are circulating. In particular, books from R. V. Kadison and J. Ringrose, from M. Takesaki and from G. K. Pedersen are expected shortly.

HERBERT HALPERN

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Principles of optimal control theory, by R. V. Gamkrelidze, Mathematical Concepts and Methods in Science and Engineering, Vol. 7, Plenum Press, New York, 1978, xii + 175 pp.

The present book is an account of lectures given at Tblissi State University. It is an excellent exposition of mathematical principles underlying Optimal Control Theory. In particular the author derives the maximum principle and establishes basic existence theorems for a special but typical optimal control problem. One of the outstanding features of the book is the clear presentation of the concept of relaxed or generalized controls which play such an important role in Optimal Control Theory.

Because the author is interested mainly in presenting basis principles underlying optimal control theory, he restricts himself to a discussion of the time optimal control problem in its simplest form. Specifically, he seeks a control

$$u(t) \qquad (t_1 \leqslant t \leqslant t_2)$$

which transfers a given initial point $x(t_1) = x_1$ to a fixed terminal point $x(t_2) = x_2$ via the motion

$$\dot{x} = f(t, x, u)$$

in the least time $T = t_2 - t_1$. For each t, the control $u(t)$ is restricted to lie in a prescribed set U in an r-dimensional Euclidean space. Of course, this is not the most general optimal control problem. However, it is of sufficient generality so that it yields basic concepts which are useful in the study of more general situations. In particular it enables the author to introduce the important concept of relaxed controls. It should be noted that when U is open, we have a version of the classical problem of Mayer in the calculus of variations.

Gamkrelidze considers two types of controls, namely, ordinary controls and relaxed or generalized controls. By an ordinary control is meant a bounded measurable function $u(t)$ $(t_1 \leqslant t \leqslant t_2)$ having $u(t)$ in U for each t. A weakly measurable finite family of probability measures μ_t concentrated on U is a generalized control. Ordinary controls can be identified with Dirac measures. A generalized control is a suitable weak limit of a sequence of ordinary controls. Thus relaxed controls are obtained by an appropriate completion of the space of ordinary controls. The author gives an excellent exposition of the theory of relaxed controls and its applications to optimal control theory.

The maximum principle is first established for generalized controls and then specialized to ordinary controls. Similarly, existence theorems are developed first in terms of generalized controls and then specialized to ordinary controls with the addition of suitable hypotheses. To carry out these developments, the author establishes appropriate existence and continuous dependence theorems for differential equations together with a theory of variations for differential equations, generalized controls, and trajectories. As a corollary he obtains an existence theorem for the simple fixed endpoint problem in the calculus of variations.

In writing this book Gamkrelidze assumes that the reader is already familiar with the elements of optimal control, including the principles of linear systems and convex set theory. He assumes also that the reader has mastered the basic notions of general measure theory. For examples and motivation he refers to the classic book *The mathematical theory of optimal processes* by L. S. Pontryagin, V. G. Boltyanski, R. V. Gamkrelidze, and E. F. Mishenko, Wiley, New York, 1962. Gamkrelidze contends that once the reader has mastered the material in the present book, the transfer to problems with more general boundary conditions and more general functionals can be accomplished, as a rule, without any particular difficulty and, in most cases, in an automatic way. This assertion is based on the assumption that the reader is already familiar with the additional phenomena and intricacies which arise when general boundary conditions and functionals are considered.

A sequel to the present book would be welcome. One which develops analogous results for general functionals and general boundary conditions including the bounded state variable case. It would be instructive also to develop a similar theory using generalized controls for problems whose solutions are not minimizing but become minimizing when suitable isoperimetric conditions are adjoined. Many problems in mechanics and in the theory of geodesics are of this nature. It should be noted that most examples appearing in the literature deal only with ordinary controls. Perhaps this is because their solutions involve only ordinary controls. There are, of course, examples which require generalized controls. As has been pointed out by E. J. McShane there is a need for developing techniques for solving typical examples from the point of view of generalized controls even when the solutions are given by ordinary controls. The remarks of McShane are given in *The calculus of variations from the beginning through optimal control theory* appearing in Optimal Control and Differential Equations, Academic Press, 1978, edited by A. B. Schwarzkoff, W. G. Kelley, and S. B. Eliason.

The book by Gamkrelidze is an important and welcome contribution to the literature on optimal control theory.

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Extremal graph theory, by Béla Bollobás, London Mathematical Society Monograph No. 11, Academic Press, London, New York and San Francisco, 1978, xx + 478 pp.

Problem Solving holds an awkward place in mathematics. Everyone agrees it is fun but some question its importance. If Mathematics is the building of a castle of Theory then there is perhaps little place for the solution of an individual problem. Yet, for others, the solution of problems is far more than an amusing pastime. The creation and solution of problems determine the direction of mathematical thought. Which of these is the correct view? An easy answer is, of course, both. But the relative importance given to these not necessarily antagonistic viewpoints helps determine the nature of our subject.

Problem Solving has long played a vital role in Graph Theory. This has led to a certain subjectivity regarding the importance of any particular result. There are a myriad of possible problems and papers flood into the already overcrowded journals. Recognition of meaningful work becomes difficult but it is not impossible. With the passage of time the main currents of Graph Theory become clearly marked and the separation of the important from the mundane may begin.

Extremal graph theory is an important addition to the Graph Theory literature. There is a staggering amount of material here. Throughout, theorems are treated not as isolated results but as part of a cohesive whole.