There are no exercises in the book unless one counts the "proof left to the reader" type. Indeed, a good beginning on a set of exercises might include the above cited examples, and others, with comments and, perhaps, hints.

Some errors are noticeable although certainly not in any great number. In the construction of a Euclidean plane on p. 18 K is assumed to be an arbitrary field yet k is chosen to be an element of K such that $-k$ is not a square. Clearly k cannot be quadratically closed. This same error is repeated on p. 23.

In Theorem 4.2 the existence of a line g is implied by the statement of the theorem; yet the proof of the theorem seems to assume that g exists.

There are relatively few typographical errors, a remarkable feat considering the complexity of some notation and the abundance of subscripts.

This book is a moderately good addition to the literature; its good features outweigh its shortcomings. It should be accessible to patient and persistent beginners and no doubt will be a valuable source for future work on the geometric theory of S-groups.

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Shape theory, by Jerzy Dydak and Jack Segal, Lecture Notes in Math., vol. 688, Springer-Verlag, Berlin-Heidelberg-New York, 1978, vi + 150 pp., $11.80.

Shape theory has come to loom large on the horizon of topology. The literature in the area has grown enormously. More and more research papers assume that the reader is familiar with the results and techniques of shape theory. For the reader who does not have this familiarity, but who wishes to learn, there are difficulties. He may struggle through a paper only to find that the results are superseded by more powerful and completely different techniques. Some results have "standard" errors which may or may not be corrected in the literature. What the newcomer will probably find most irritating is the teeming multitude of approaches to shape theory that he will find. Each approach is derived from a particular viewpoint according to the whim of its originator. Some of these approaches are confused and capable of permanently beclouding the mind as the searcher seeks to find the depth that is not there. Some approaches are so abstract that even experienced mathematicians marvel in wonder at the meaning of it all. To those who are experienced in the area, there is an approach to shape theory which has come into focus as the likely vehicle to carry this knowledge to future generations of mathematicians. This approach makes use of category theory and a few specific categorical constructions. The pro-category construction and the Kan or Čech extension of a functor are the principal categorical tools although they are somewhat disguised in the theory. The advantage of this viewpoint is that the definitions necessary to shape theory can be stated clearly and precisely quite easily. Furthermore, as one moves from the category of compact metric spaces to the category of "bicompacta", to the category of pointed spaces, to the category of locally compact spaces in "proper shape theory", the definitions change ever so slightly and the whole field takes on a unified view. The newcomer must still be patient in his pursuit of this esoteric knowledge, but with persistence the vista will open wide in a vast panorama.

Dydak and Segal in the volume under review have given us an excellent introduction to shape theory. It not only presents the principal results of the theory, but it gives us the perspective and unified view which gives shape theory its greatest appeal.

According to the view of the authors, shape theory is due to Karol Borsuk. In the late sixties Borsuk suggested an approach for studying metric compacta which allowed one to ignore any local pathology which may exist in the compactum and see it from a global view. His approach was to embed the compactum in the Hilbert cube or another appropriate metric absolute retract. One then studies not the compactum itself, but its system of neighborhoods. This has to be done by comparison with the embeddings of other compacta. The comparison is done by means of specially defined morphisms between embedded compacta. Borsuk's approach caught the imagination of topologists. It was exciting to be studying the geometry of spaces that previously didn't seem to have any geometry at all.

On the other hand, the authors may have presented too narrow a view of the history of the subject. You might say that there were glimpses of shape theory from the very beginnings of modern topology. Work by Alexandroff, Čech, Vietoris, and others attempted to approximate arbitrary topological spaces by simplicial complexes in one fashion or another. This is the basic idea of shape theory. It seems that at that time they lacked the mathematical technology which has given shape theory its success. Many of the contributions to shape theory have been not so much to discover an original idea as to recognize work done in other areas of mathematics as being relevant to the purposes of shape theorists. It may not be possible to ever disentangle all these interrelationships and establish the true origins of all the ideas involved. However, even the beginner should realize that shape theory is not an isolated event in the stream of ideas. It should rather be recognized as a focal point and Borsuk's contribution should be seen as one of a catalysis that stimulated the convergence of these ideas at a critical time.

The categorical approach which the authors have adopted has one particularly appealing feature. It provides a clear analogy with homotopy theory. One could probably call shape theory "Čech homotopy theory". The analogy between homotopy theory and shape theory is similar to that between singular homology theory and Čech homology theory. The analogy has led to shape versions of many of the theorems of homotopy theory: the Whitehead theorem, the Hurewicz theorem, the Vietoris theorem, and others. It has led to new concepts which generalize the notion of absolute neighborhood retract (movability and related ideas). It has given new uses for and insights into derived limits. In particular, Lim^1 has played an important role in both theories. And, it has led to the use of new algebraic invariants for topological spaces, the pro-homology groups and the pro-homotopy groups. (These are *not* groups by the way.) These topics are all introduced in this volume with a guide to the literature for those who wish to pursue any topic in greater depth.

Shape theory is not simply a generalization of homotopy theory, nor does it deal only with hopelessly complicated examples. One of the beautiful theorems of the theory is Chapman's Complement Theorem. *Shape theory* gives a detailed account of this theorem for metric compacta. Chapman's theorem states that one can associate with each metric compactum X a connected Q-manifold M_X such that X and Y are shape equivalent if and only if M_X and M_Y are homeomorphic. Actually, M_X is just the complement of X in the Hilbert cube if X is embedded in one of the faces of the cube. This allows one to associate with X an object which is very geometric and which contains all the information about the shape of X. It is a bridge between infinite dimensional topology and shape theory.

One drawback of *Shape theory* is that there are not enough examples. In any theory there are not only theorems, but standard counterexamples that provide insight and motivation for those in the field. The beginner needs to see not only major landmark examples some of which are included here, but also more of the homely variety as well. Beyond this there are many basic counterexample constructions that must be mastered for a proper understanding of the theory which are not included here.

One has to be a topologist to appreciate the types of spaces one contemplates in shape theory. However, one does not need to be a topologist to encounter such spaces and have need to analyze them. One may encounter a continuum in R^n which is the intersection of polyhedral neighborhoods. Unfortunately, this may be any continuum in R^n and it may be very pathological indeed even if the polyhedral neighborhoods are n-cells. The Stone-Čech compactification is encountered in various guises in all fields. It is so pathological that only the truly initiated appreciate how complicated it really is. Many algebraic objects have structure spaces associated with them. These structure spaces are often impossible to visualize even though the basic construction can be easily stated. One may encounter a nice mapping between reasonable spaces. However, the point inverses of such a mapping may not be reasonable at all. Could we get along without compact topological groups? Even though these groups are the inverse limits of Lie groups they may be quite difficult. Solenoids, for example, take some getting used to. You may shudder to think that there are compact groups more complicated than solenoids. Shape theory has had some success in dealing with these examples. If you have encountered similar examples and refuse to panic, shape theory may be of some help and *Shape theory* can help get you started on the right foot.

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