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Lehrbuch der Mathematischen Physik, vol. 3, *Quantenmechanik von Atomen und Molekülen*, by W. Thirring, Springer-Verlag, Vienna-New York, 1979, x + 263 pp., $19.80.

Mathematical physics is a remarkable enterprise on the borderline between two intellectual disciplines. Its practitioners do mathematics, of course, though they are inclined to deny this when confronted by their physicist colleagues. But, unlike in pure mathematics, the subjects and problems in mathematical physics are determined, at least to some extent, by extra-mathematical considerations, such as relevance (however tenuous occasionally) to some aspect of physics, or the desire to explore mathematical phenomena whose very existence is suspected only because of the belief that this or that mathematical model describes some empirically known phenomenon. But how does one learn mathematical physics? A mathematician, with professional knowledge of the basics in a number of areas of modern mathematics, may decide to learn physics. A more usual pattern is that one knows physics to start with, and a desire to do things rigorously, to search for precision and logical clarity, to look for simplicity and well-defined structures, leads one to learn the mathematics relevant to a particular physical subject gradually. In either case, one of the difficulties for the novice is the virtual absence of physics textbooks that take the point of view of mathematical physics seriously from the beginning. There is the research literature, of course, and there are "mathematical methods" books. Both are useful, but do not fill the needs of those who may want to learn physical theory, but insist on the precision and clarity that only mathematics allows. For these reasons the appearance of Thirring's book is a very welcome addition to the literature on quantum mechanics. That subject is one of the great intellectual creations of our century, and is the foundation for the understanding–such as it is–of the structure of matter. No wonder that books on it pour into major university libraries at a rate that at times seems to exceed their capacity to catalogue them. But much of this literature is repetitious, and hardly any improvement over what was available twenty, indeed forty years ago. But the Thirring book is a new departure. It is a text on quantum mechanics, yes; but it is written fully in the spirit of mathematical physics, and incorporates much interesting research done in the last twenty years. It is the third volume in a series, the first two of which appeared earlier, with a fourth volume promised, and the set, when complete, will represent a modern *Course of theoretical physics* as a worthy successor of classics of an earlier era such as the Sommerfeld series, the lectures of Pauli, or the Landau-Lifschitz series. Of course, not everyone will want to learn quantum theory (or any other subject of physics) with such emphasis on mathematical rigor, perhaps not even mathematicians, but for those who do there is now a text fully satisfying their needs.

Quantum mechanics, among other physical subjects, is peculiarly suited for a treatment that insists on the precision and, more importantly perhaps, on the spirit of contemporary mathematics. For this there are actually two

reasons. First, in contrast with classical theories, the very concepts with which quantum physics operates are against common sense and therefore partake of an abstraction that needs mathematical formulation (observables identified with selfadjoint unbounded Hilbert space operators; waves propagating in $3n$, instead of 3, dimensional space; complex probability amplitudes: just to mention a few of a host of concepts far removed from *Anschaulichkeit* and typical of quantum theory). Thus to clarify the perplexing and seemingly paradoxical features of atomic physics, a high degree of mathematical abstraction is already needed at the foundations. Secondly, quantum mechanics gives rise to many problems in Analysis which have a surprisingly rich and intricate nature, whose naive or formal treatment can easily result in erroneous conclusions. Worse even, many times naive and oversimplified mathematics gives correct results, but then the problem is to explain rationally why this happens. In any case, the full depth of even simple quantum mechanical problems can only be appreciated by a sophisticated and careful mathematical treatment. But, above all, perhaps the main justification to do theoretical physics rigorously is the desire to know. Paraphrasing two much respected practitioners of quantum mechanics who once wrote "Mathematics is an interesting intellectual sport, but should not be allowed to stand in the way of obtaining sensible information about physical processes", may I be allowed to say that, while obtaining sensible information about physical processes is an interesting sport, it should not be allowed to stand in the way of the distinction between knowing what follows from what and only believing, however strongly. The reader of Thirring's "Quantenmechanik" will not fail to understand that distinction if he is sympathetically inclined to its spirit.

From the point of view of contents, the organization of the book is fairly conventional. It can be divided into roughly two parts; the first 150 pages devoted to general theory, the last 100 to particular problems of atomic physics. On all of the general subjects discussed, from the conceptual foundations of quantum theory, through unbounded operators, commutation relations, rotation invariance, to the Schrödinger equation and its scattering and perturbation theory, one finds a wealth of mathematical detail, handled rigorously, illustrated judiciously, and explained tersely. The most interesting part of the book is Chapter 4. This takes up six important problems in atomic physics (the H-atom, the H-atom in external fields, the He-atom, scattering on simple atoms, complex atoms, simple molecules). The emphasis here is on calculations. But calculations in a very different spirit from the traditional concept of "calculations" by theoretical physicists of the past. As the author states in the Preface, he has endeavored replacing the usual computations with their uncertain accuracy by error bounds, so that the raw habits of theoretical physicists shall be refined to the more cultured ones of their experimental colleagues (" . . . um die rauhen Sitten der theoretischen Physik zu den kultivierteren der Experimentalphysik zu verfeinern."). Essentially, this means adopting the spirit of modern numerical analysis to the great task of extracting concrete, quantitative, and reliable information from the theory. This is not just a matter of error bounds. Sometimes mathematical meaning must be given to computations which, on the surface of it, are inconsistent. Here is one striking example. The negative spectrum of the energy operator

for the H-atom is a point spectrum. When the H-atom is in an electric field, no matter how weak, the whole spectrum is continuous. Yet "energy eigenvalue shifts" are calculated by perturbation theory. Quite apart from the fact that the results are verified experimentally to the last decimal point, what is the meaning of this? I will not deprive the reader of the pleasure of seeing this paradox removed; be it enough to say that it is only a sample of many problems whose correct treatment, found in the book, requires sophisticated and correct mathematics.

Perhaps even more than its content, the style and small scale organization of the book is worthy of remark. Typically, there is a short definition, followed immediately by a series of examples, and then a numbered series of remarks. Pedagogically speaking, the heart of the book is in these remarks. They are intended not just to enlighten, to put forth additional facts, to establish cross connections with topics discussed elsewhere, but they are also intended to warn what *not* to think, what misunderstanding to avoid, where and in what way a naive point of view is inadequate. Then there may come a short expository paragraph or two, followed by a collection of problems. The solutions to the problems are given in full immediately afterwards, in enough detail so that they can be read as part of the main text. A theorem is stated perhaps, followed by three or four remarks, then proof, then more remarks. Sometimes we encounter a paragraph entitled Necessary and Sufficient Conditions for this or that state of affairs; or a paragraph entitled Clarification; or a counterexample for a plausible but untrue proposition. All this is written at a fast clip, in a pleasant, straightforward, occasionally even colloquial German ("Man könnte daran denken . . . dabei kann man aber Schiffbruch erleiden"), far from a cut-and-dry overpolished presentation, and while proper mathematics is insisted on, there is a feeling of excitement and enthusiasm between the lines.

Unfortunately, I cannot conclude this review with praise only. This may be a small matter, but it is disturbing nonetheless. The printing job is poor. This is the more surprising because one is used to the high standards of the Springer-Verlag. Perhaps the explanation is that Springer-Austria has not the same typographical means at its disposal as does Springer-Germany, or that in order to keep the price reasonable (which it is), radical sacrifices were made in printing quality. Be that as it may, one wishes that a work of this nature could see the light in a visually at least satisfactory edition. Here are some of my complaints: mathematical symbols are not italicized, whether displayed or in the running text, resulting in a general strain on the eye. Built up fractions are aligned as if they had been produced on the typewriter. The arrow for vector, mapping, and limit looks the same. There is no apostrophe, cnly a raised comma, so that second derivatives and double duals look like the end of a quotation. Some may disagree, but I prefer language rather than symbol where there is no ambiguity; but in any case there is no excuse to use a capital V with the minus sign superposed to stand for the universal quantifier. Let this be enough, and let me conclude with the hope that if there will be a second edition, it will be possible to pay more attention to this regrettable blemish on this otherwise interesting, valuable, and pioneering quantum mechanics text.

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